

DUAL-MEMBER MIXED PROPORTIONAL: A NEW ELECTORAL FORMULA FOR

CANADA

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## CONTENTS

ACKNOWLEDGEMENTS	4
<b>1</b> ABBREVIATIONS	4
<b>2</b> DEFINITIONS	4
<b>3</b> INTRODUCTION	5
<b>4</b> THE IDEAL ELECTORAL FORMULA	6
<b>5</b> CRITIQUES OF PROMINENT ELECTORAL FORMULAE	9
<b>5.1</b> SINGLE MEMBER PLURALITY	9
<b>5.2</b> MIXED MEMBER PROPORTIONAL	13
<b>5.3</b> SINGLE TRANSFERABLE VOTE	16
<b>5.4</b> CONCLUDING REMARKS	19
<b>6</b> DUAL-MEMBER MIXED PROPORTIONAL	20
<b>6.1</b> THE BASICS	20
<b>6.1.1</b> AN EXAMPLE	22
<b>6.1.2</b> NOTES ON EXAMPLE	28
<b>6.1.3</b> GENERAL NOTES ON THE DESIGN OF DMP	29
<b>6.2</b> BALLOT DESIGN	30
<b>6.3</b> THE EXCEPTION: INDEPENDENTS	31
<b>6.4</b> THE THRESHOLD	31
<b>6.5</b> ADDRESSING TIED VOTES	33
<b>6.6</b> BY-ELECTIONS	36
<b>6.7</b> WHEN DMP FAILS TO PRODUCE PROPORTIONAL RESULTS	37

<b>6.7.1</b>	CASE 1: ONE OR MORE PARTIES WINS A FRACTION OF THE FIRST SEATS GREATER THAN TWICE THEIR FRACTION OF THE POPULAR VOTE	37
<b>6.7.2</b>	CASE 2: ONE OR MORE PARTIES WINS FEWER SEATS THAN THEY DESERVE BECAUSE THEY DID NOT RUN A FULL SLATE OF CANDIDATES	38
<b>7</b>	CONSTITUTIONAL ADHERENCE	39
<b>8</b>	GERRYMANDERING	40
<b>9</b>	TESTING DMP IN MATLAB	41
<b>9.1</b>	INTRODUCTION	41
<b>9.2</b>	PAST FEDERAL ELECTION OUTCOMES WITH DMP	42
<b>9.2.1</b>	OVERVIEW OF RESULTS	42
<b>9.2.2</b>	REGIONAL REPRESENTATION	43
<b>10</b>	THE CORRECTION FACTOR AS A STRENGTH OF DMP	46
<b>11</b>	CHALLENGING ACCEPTED LIMITATIONS OF ELECTORAL FORMULAE: WHY DMP COULD BRIDGE THE GAP IN THE ELECTORAL REFORM DEBATE	48
<b>12</b>	CONCLUSION	49
	AFTERWARD	50
	APPENDIX A: CALCULATING THE MAXIMUM POTENTIAL FOR WASTED VOTES WITH PROPORTIONAL FORMULAE	51
<b>A.1</b>	THE NUMBER OF PARTIES IS LESS THAN TWICE THE NUMBER OF SEATS	52
<b>A.1.1</b>	THE NUMBER OF PARTIES IS EVEN	52
<b>A.1.2</b>	THE NUMBER OF PARTIES IS ODD	53
<b>A.2</b>	THE NUMBER OF PARTIES IS GREATER THAN TWICE THE NUMBER OF SEATS	54
	APPENDIX B: BALLOT DESIGN	55

APPENDIX C: 2004 FEDERAL ELECTION	57
APPENDIX D: 2006 FEDERAL ELECTION	61
APPENDIX E: 2008 FEDERAL ELECTION	65
<b>E.1</b> FEDERAL RESULTS	65
<b>E.2</b> SELECT PROVINCIAL RESULTS	68
APPENDIX F: 2011 FEDERAL ELECTION	71
<b>F.1</b> FEDERAL RESULTS	71
<b>F.2</b> SELECT PROVINCIAL RESULTS	74
APPENDIX G: THE DISTRICT REPRESENTATION FACTOR	77
REFERENCES	79

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## 1 ABBREVIATIONS

**DMP** – Dual-Member Mixed Proportional

**MMP** – Mixed Member Proportional

**OPOV** – One-Person One-Vote

**SMP** – Single Member Plurality

**STV** – Single Transferable Vote

**TMP** – Two Member Plurality

## 2 DEFINITIONS

**Correction Factor** – The fraction of seats that are assigned differently under DMP than under a two-member district plurality system.

**District Representation Factor** – The number that when multiplied by a party's fraction of the seats in an elected assembly yields the fraction of the districts in which the party has representation.

**Electoral Formula** – An algorithm that takes votes as an input and produces winning candidates as an output.

**Electoral System** – Refers to the combination of ballot design, districting, and electoral formula.<sup>1</sup>

**Margin of Error** – The difference between a party’s fraction of the popular vote and its fraction of the seats in an elected assembly.

**OPOV** – The principle that each member of the electorate has one vote and that those votes count equally to determining election outcomes.

**TMP** – An electoral formula that functions identically to SMP but in dual instead of single member districts.

### 3 INTRODUCTION

In the last decade, electoral formula reform has garnered serious attention in Canada.<sup>2</sup> Indeed, multiple Canadian governments have considered changing their electoral formulae to better reflect the will of the electorate (i.e., to produce more proportional election outcomes).<sup>3</sup> Three provinces (British Columbia, Ontario, and Prince Edward Island) have held referenda on adopting alternative electoral formulae, two provinces (Quebec and New Brunswick) have studied electoral formula reform, and the federal government has established a commission that has recommend adopting a new electoral formula.<sup>4</sup> However, despite the extensive discussion on the subject and four referenda, electoral formula reform has not been achieved.<sup>5</sup>

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<sup>1</sup> Dennis Pilon, *The Politics of Voting: Reforming Canada’s Electoral System* (Toronto: Emond Montgomery Publications Limited, 2007), 18.

<sup>2</sup> Pilon, *The Politics of Voting*, 89.

<sup>3</sup> The components of an electoral system are not isolated from one another, and changing one often necessitates changes in the others. What is important to note here is that in considering changing their electoral formulae, these governments have contemplated changing their respective jurisdiction’s entire electoral system.

<sup>4</sup> “Electoral Reform Initiatives in Canadian Provinces,” last modified August 18, 2009, <http://www.parl.gc.ca/Content/LOP/ResearchPublications/prb0417-e.htm>; Pilon, *The Politics of Voting*, 89-90.

<sup>5</sup> “Electoral Reform Initiatives in Canadian Provinces.”

Reformists agree that Single Member Plurality (SMP) is flawed and should be replaced, but they have not been able to agree on a replacement formula so far. The most commonly suggested alternatives to SMP are Single Transferable Vote (STV) and Mixed Member Proportional (MMP), with MMP being the favourite of the federal government and the provinces that have considered electoral formula reform.<sup>6</sup> These are presented as worthy alternatives to SMP despite having significant flaws. Although these flaws are not necessarily massive enough to make these alternative formulae inferior to SMP, their existence merits going back to the drawing board to develop a better alternative.

The aim of this report is to present that better alternative. Like many works on electoral reform, an explanation of why SMP is flawed and needs to be replaced will be provided. However, unlike many other writings on this subject, both STV and MMP will be thoroughly criticized as well. The collective flaws of these three formulae will inform the parameters for the construction of a new alternative, Dual-Member Mixed Proportional (DMP). The remainder of the report will be devoted to explaining and defending this new formula. However, to begin this project, a clear target at which all electoral formulae should aim must be established.

#### 4 THE IDEAL ELECTORAL FORMULA

Before examining how SMP and its most discussed potential replacements are inadequate, it is necessary to describe what an electoral formula should accomplish. These terms of reference will serve as the ideal that all electoral formulae should strive to achieve and will create an objective scale with which to compare different electoral formulae. While any list of objectives will be open for debate, I contend that the following six objectives are the most defensible as an outline of what the ideal electoral formula should achieve. First, it should produce proportional

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<sup>6</sup> “Electoral Reform Initiatives in Canadian Provinces”; Pilon, *The Politics of Voting*, 89-90, 119. PEI, New Brunswick, and Ontario have recommended MMP, while BC has recommended STV.

results. In other words, an electoral formula should translate parties' fractions of the popular vote into the same fractions of the seats in an elected assembly with a margin of error no larger than  $\frac{1}{\text{total number of seats}}$ . Second, an electoral formula should achieve One-Person One-Vote (OPOV). That is to say, each person's vote should count equally when determining election outcomes. Third, an electoral formula should only make use of first choice preferences. Fourth, electoral formulae should waste as few votes as possible, meaning that formulae should maximize the fraction of votes that count towards electing representatives.<sup>7</sup> Fifth, an electoral formula should not breed intra-party competition. Finally, an electoral formula should permit the distribution of an elected assembly's seats across as many districts as possible, which requires that it minimize the number of representatives allocated to each district.

The above criteria are not entirely distinct since there are two logical connections that exist. The first is between criteria two and three. If an electoral formula adheres completely to OPOV, it will only count first choice preferences. This is the case because to not count the first choice of every voter is to create inequality between votes—electing one's first choice preference is not the same as electing one's fourth, or even second. However, an electoral formula can exclusively count first choice preferences but still fail to achieve OPOV. Therefore, strictly limiting an electoral formula to counting first choice preferences is a necessary, but not a sufficient, condition for OPOV. The second logical connection exists between criteria one, two, and four. If an electoral formula produces proportional outcomes, each vote will have been counted equally and, therefore, OPOV will have been achieved. Furthermore, criterion four will be satisfied since as many votes will have a direct impact on the results as possible. Of these

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<sup>7</sup> Appendix A explains how to calculate the maximum fraction of votes that can be wasted by a proportional electoral formula.

three criteria, the only one that can be satisfied independently is OPOV.<sup>8</sup> However, as will be explained in section five, accomplishing this is virtually impossible.

The ideal electoral formula has been defined in the above manner for four reasons. First, it is fundamental to democracy to ensure that the majority rules. This is impossible if the electorate is given differential voting power or if one party has its support arbitrarily inflated from a minority position to complete legislative control. Criteria one through four ensure that the will of the majority will be reflected accurately in an elected assembly. Second, criterion one, which has been deemed necessary to adhere to democratic principles, requires the existence of political parties. Without these vehicles of representation, proportional representation would have no means of coming into existence. Therefore, political parties must remain effective instruments of representation. Fostering internal bickering and competition would not serve this purpose; thus, criterion five has been introduced to maintain the integrity of political parties. Third, the electorate must be able to make informed decisions. Criterion six strives to ensure that this is possible by dividing the responsibility for choosing the members of an elected assembly between as many districts as possible. This reduces the number of candidates that each district has to choose from and, as a result, reduces the effort required by voters to become informed. Fourth, an elected assembly should be cognizant of the issues facing all areas it is responsible for governing. By separating the community into as many regions as possible (given the agreed upon size of the elected assembly), criterion six attempts to prevent the members of an elected assembly from disproportionately coming from one region.

Implicit in this justification is that an electoral formula must foster accountability. The first step to creating an environment where politicians can be held accountable is to actually elect the

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<sup>8</sup> For example, an election with two districts, 10 votes cast in each district, and that is to be decided by SMP would achieve OPOV but would not produce proportional results or waste as few votes as possible by design.

politicians and parties that the electorate have chosen. A proportional electoral formula accomplishes this task. Equally important to ensuring accountability is developing an electoral formula that allows voters to make informed decisions. By dividing the responsibility of electing members to an assembly across as many districts as possible, the number of candidates that voters have to choose from is reduced to as small a number as possible. As a consequence, the time and effort required for a voter to become informed about their local candidates is minimized and, therefore, the possibility of them becoming informed is maximized.

## 5 CRITIQUES OF PROMINENT ELECTORAL FORMULAE

### 5.1 SINGLE MEMBER PLURALITY

From its name, one can discern that SMP is not a proportional electoral formula. The fact that it uses pluralities to elect representatives in single-member districts means that there is no safeguard built into the calculation to ensure a proportional outcome. Canada's history has shown that the absence of an electoral formula that explicitly works to produce proportional results makes achieving proportionality highly unlikely; indeed, the SMP electoral formula has produced disproportionate results in every federal election since Canada's inception.<sup>9</sup> Even more concerning, though, is that this lack of proportionality in the results produced by SMP has made false majority governments far more common than true majority governments.<sup>10</sup> Since 1921, Canada has had thirteen false majority governments but only three true majority

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<sup>9</sup> "Canadian Election Results by Party: 1867 to 2008," accessed August 2, 2013, <http://www.sfu.ca/~aheard/elections/1867-present.html>; "Electoral Results by Party," last modified October 7, 2011, <http://www.parl.gc.ca/parlinfo/compilations/electionsandridings/ResultsParty.aspx>.

<sup>10</sup> A false majority government is one that has over 50% of the seats in the House of Commons but less than 50% of the popular vote (Russell 2008, 10).

governments.<sup>11</sup> This is a problem for a country that claims to be a democracy, as it is fundamentally undemocratic to translate a minority of votes from the electorate into a majority of seats in the House of Commons. For this reason, the lack of proportionality in the results produced by SMP is its most significant flaw.

Another democratic failing of SMP is its dismal record on voter equality. There are three reasons why SMP fails to achieve OPOV in Canada. First, the minimum seat allocations to each province and territory outlined in sections 51(1) rule 2, 51A, and 51(2) of the 1867 Constitution Act prevent the proportional distribution of federal seats.<sup>12</sup> For example, John Courtney points out that the provincial averages for the population of electoral districts varied from 33,824 in Prince Edward Island to 108,548 in British Columbia in the post-2001 redistribution.<sup>13</sup> In this redistribution, PEI only deserved one seat based on its population but was allocated four because it is constitutionally guaranteed four seats.<sup>14</sup> This meant that votes cast in PEI were, on average, worth 3.2 times as much as those cast in BC.<sup>15</sup> Second, districts are permitted to vary in size within the provinces. Under normal circumstances, the populations of districts within a province are allowed to be 25% larger or smaller than the provincial electoral quotient, which is “a province’s population divided by its number of seats.”<sup>16</sup> A calculation of the Gini Index has shown that this leeway has been utilized when electoral districts have been drawn, as no province has had exact equality in district size during the hundred-year period the index was calculated.<sup>17</sup> Third, if the absolute (not relative) voter turnout is not equal from one district to the next, then

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<sup>11</sup> “Electoral Results by Party”; Peter Russell, *Two Cheers for Minority Government: The Evolution of Canadian Parliamentary Democracy* (Toronto: Emond Montgomery Publications Limited, 2008), 10.

<sup>12</sup> “Constitution Act, 1867,” last modified August 30, 2013, <http://lois.justice.gc.ca/eng/Const/page-2.html#docCont>.

<sup>13</sup> John C. Courtney “Canada’s Electoral System,” in *Canadian Parties in Transition, third edition*, ed. Alain-G. Gagnon et al. (North York: University of Toronto Press Incorporated, 2011), 287.

<sup>14</sup> Courtney, “Canada’s Electoral System,” 285-6.

<sup>15</sup> This assumes that the relative provincial voter turnouts were equal.

<sup>16</sup> *Ibid.*, 287.

<sup>17</sup> *Ibid.*, 288. Note that the Gini index was calculated for the redistributions that occurred between 1903 and 2003.

votes cast in different districts will not be of equal weight. For example, consider two districts that each have a voting population of 10. If the first district has 100% voter turnout and the second 50%, the votes cast in the second district would be worth twice as much as those cast in the first. Therefore, although these hypothetical districts have been drawn to achieve OPOV, the voter turnout has thwarted the effort.

Related to the issue of OPOV is the problem of wasted votes. Allowing pluralities of votes to decide election outcomes permits a majority of votes to not count towards the election of any candidates or, in other words, to be wasted. For example, consider a single-member district that has ten candidates contesting the election. If the first candidate receives 10.9% of the vote and each of the other nine candidates receives 9.9%, the first candidate would win the seat.

Therefore, 89.1% of the votes cast in this district would not contribute to the election of a representative, or, to say it differently, 89.1% of the voters would fail to achieve representation. At this point, it may be tempting to say that this is just a theoretical problem that has no practical implications, but the evidence suggests otherwise. For the federal elections that occurred between 1980 and 2006, an average of 49% of the votes cast were wasted.<sup>18</sup>

The issue of wasted votes leads directly to the problem of strategic voting. Strategic voting is a behaviour engaged in by some voters when their party of choice is likely to place third or lower and their least favourite party is perceived to be in the lead. Since it is thought that a vote for their preferred party is guaranteed to be wasted, such voters decide to vote for the perceived runner-up, with the hope that their votes will put them in the lead. Rather than aim to be represented by their party of choice, these voters attempt to prevent being represented by their least favourite party. This behaviour is a concern because it is indicative of a sentiment in some voters that their voices will not be heard if they cast a vote for their preferred party. In a

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<sup>18</sup> Pilon, *The Politics of Voting*, 54.

democracy, citizens must be able to be represented by the parties of their choice. Therefore, to use a system that produces representation that the voters do not want is not only illogical, it is also undemocratic.

Yet another criticism of SMP is that it exacerbates regional differences. An analysis of elections that occurred between 1968 and 2000 found that SMP

turned national parties with some regional bias in their support base (such as Reform/Canadian Alliance) into purely regional parties that are thenceforth perceived and attacked as such. For governing parties, support in key regions has been exaggerated (most recently, federal Liberals in Ontario 1993-2005), with its own distorting effects on the party and the government it formed. The representation of certain provinces within the winning party caucus has been inflated, reinforcing their demographic weight in Canadian political life. With the partisan heterogeneity of provinces artificially reduced, diverse provincial electorates have been simplistically and misleadingly compressed into a single partisan voice.<sup>19</sup>

Therefore, it can be argued that SMP is detrimental to Canadian unity, since it “frequently turns a national electorate only moderately divided along regional lines into a regionally fractured and polarized House of Commons.”<sup>20</sup>

Finally, SMP makes drawing district boundaries a contentious issue. Since districts are often won with a plurality of votes rather than a majority, a slight change in how votes are cast can change the outcome. Therefore, a minor shift in a district’s boundaries could be enough to change the party that will garner a plurality of votes. This opens the door to gerrymandering, both real and perceived. Terence Qualter demonstrates that some types of voter distributions lack an objective “right way” to draw district boundaries.<sup>21</sup> Consequently, regardless of the

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<sup>19</sup> James Bickerton, “Between Integration and Fragmentation: Political Parties and the Representation of Regions,” in *Canadian Parties in Transition*, third edition, ed. Alain-G. Gagnon et al. (North York: University of Toronto Press Incorporated, 2011), 414.

<sup>20</sup> *Ibid.*, 430.

<sup>21</sup> Terence Qualter, *The Election Process in Canada* (Toronto: McGraw-Hill Company of Canada Limited, 1970), 115-116.

drafting process, “the drafters will be charged with gerrymandering by whichever party feels that it has not got out of the scheme all that it expects.”<sup>22</sup>

## 5.2 MIXED MEMBER PROPORTIONAL

MMP achieves proportionality by electing federal or regional members from party lists in addition to members that are elected in single-member districts by plurality. The federal or regional members are chosen in a manner that counteracts the disproportionate results obtained from the SMP component of the formula. Therefore, the ability of this formula to obtain proportional results is dependent on two factors. First, the more federal/regional seats (or list seats) there are in comparison to district seats, the greater the ability of MMP to produce proportional election results. Second, the ability of MMP to produce proportional election outcomes is improved as the number of regions is reduced.

Unfortunately, due to the constitutional restraints that were noted earlier, federal seats cannot currently be used in Canada. Instead, the best that can be done is to use the provinces and territories as regions.<sup>23</sup> This causes the quality of the results produced by MMP to decrease in two important ways. First, it reduces the level of proportionality in the results the formula produces. Applying MMP to the federal vote totals is not equivalent to applying the formula to provincial and territorial vote totals—even if the total number of list seats is the same. To see why this is true, consider an elected assembly that is to have 20 members elected using a proportional formula. If the 20 members are elected from one district, each party can be over- or under-represented by at most one seat. Therefore, the margin of error can be at most five percentage points. However, if the 20 members are elected from two ten member districts, each

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<sup>22</sup> Ibid.

<sup>23</sup> For a more detailed discussion of this issue, see Law Commission of Canada 2004: 84.

party can be over or under-represented by at most one seat in each district or two seats in total. This would allow the margin of error to be as high as ten percentage points.<sup>24</sup>

It should also be noted that this problem is exacerbated when the regions have unequal voter to representative ratios. For example, consider two regions, each with five seats. Region one has a population of eight, whereas region two has a population of two. If all of region one's votes go to party A, and all of region two's votes go to party B, both parties will receive five seats. However, proportionality requires that party A receive eight seats and party B two seats. Therefore, over-representing one region and under-representing another works to create disproportionate results in an MMP system.

Second, the constitutional requirements prevent MMP from achieving OPOV. Since MMP is a proportional electoral formula, it uses aggregate district results to determine election outcomes. This allows MMP to attain OPOV in each region but prevents it from meeting this objective between regions. It is not theoretically impossible for MMP to realize OPOV between regions in general, but Canada is an exception because of the disproportionate distribution of seats between the provinces that is stipulated by the constitution. However, this is not the main limiting factor. Even if the seats were distributed proportionally between the provinces, the level of voter turnout would need to be constant from one province to the next for OPOV to be realized.<sup>25</sup> Therefore, the only practical way in which MMP can be made to achieve OPOV is to have only one region.

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<sup>24</sup> Note that there is no significance to the choice to divide the number of seats evenly between the districts. The cause of the effect is the creation of the second district.

<sup>25</sup> In this case, each region would have the same voter to seat ratio but, most probably, a different total number of seats. Therefore, the relative voter turnouts would need to be equal, not the absolute, for OPOV to be achieved. If the relative voter turnouts were equal (e.g. 60%), the voter to seat ratio would be unaffected. It is the voter to seat ratios that must be equal for OPOV to be realized, which means the absolute number of voters per seat must be equal.

The final drawback of MMP is the very mechanism it utilizes to achieve proportionality, the regional seats. The use of regional seats creates two problems. The first is that it requires the single-member districts to be enlarged if the the number of seats in the House of Commons is to be maintained. This would represent a reduction in the level of local representation that currently exists in Canada. The second problem is more significant. To elect the regional seats, party lists would be required. If voters were not allowed to rank candidates on the party lists, then it would reduce the ability of the electorate to hold elected representatives accountable. However, allowing voters to rank the candidates would only marginally improve the situation. Since Canadian elections can be as short as 36 days, voters would not have much time to gather information about the regional and local candidates.<sup>26</sup> For example, in a formulation of MMP that has one regional seat for every three district seats (which would not produce proportional results in general),<sup>27</sup> Ontario could have up to 30 candidates on each party list.<sup>28</sup> It is unlikely that voters would be able to gather enough information about such a large number of candidates to make an informed decision. Furthermore, allowing voters to rank candidates from the same party would promote intra-party competition. One of the main functions of political parties is to aggregate interests. This would be difficult to accomplish if the bodies that are meant to unite like groups of interests were themselves divided. Instead of running as a team that would mutually benefit from votes for their party, candidates would have an incentive to campaign against other candidates running under the same party banner.

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<sup>26</sup> “Canada Elections Act: Part 5,” last modified June 22, 2012, <http://www.elections.ca/content.aspx?section=res&dir=loi/fel/cea&document=part05&lang=e#sec64>. See section 57.1.2.c.

<sup>27</sup> The Law Commission of Canada used a ratio of one regional seat for every two district seats and did not get proportional results (Law Commission of Canada 2004, 96).

<sup>28</sup> “House of Commons Seat Allocation by Province,” last modified March 23, 2012, <http://www.elections.ca/content.aspx?section=res&dir=cir/red/allo&document=index&lang=e>. This is based on the redistribution done in 2011.

### 5.3 SINGLE TRANSFERABLE VOTE

STV is arguably the most complicated of the electoral formulae discussed in this report. Simply put, it works by choosing winning candidates based on voter rankings in multi-member districts. Each voter is able to rank a series of candidates in their district so that their vote can be transferred to lower ranking candidates if their first choice loses or wins with a surplus of votes. However, in addition to being one of the more complicated electoral formulae, STV is also one of the more problematic. This becomes immediately clear when it is realized what is missing from STV.

When discussing STV, Rae notes that, “the party affiliations of the candidates make no difference.”<sup>29</sup> This is not a trivial fact. The trademark of a proportional formula is its recognition and use of party affiliations. This is what enables proportional formulae to translate parties’ percentages of the popular vote into equal percentages of the seats in an elected assembly. The consequence of STV’s ignorance of candidates’ party affiliations is that it lacks a mechanism to produce proportional election outcomes.

Despite the absence of a mathematical construct within the STV formula that works to produce proportional election results, it is often presented as a proportional formula.<sup>30</sup> This theoretical failure is overlooked since, it is argued, STV produces “fairly proportional” results in practice.<sup>31</sup> However, the results for the 2011 election in Ireland (one of the few countries that use STV)<sup>32</sup> depart substantially from proportionality. In this election, the first place party received a 10 percentage point boost, while the third place party was under represented by nearly

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<sup>29</sup> Douglas W. Rae, *The Political Consequences of Electoral Laws, revised edition* (New Haven: Yale University Press, 1971), 38.

<sup>30</sup> Law Commission of Canada, *Voting Counts: Electoral Reform for Canada* (Ottawa: Law Commission of Canada, 2004), 80-1; Pilon, *The Politics of Voting*, 132.

<sup>31</sup> Pilon, *The Politics of Voting*, 132.

<sup>32</sup> Law Commission of Canada, *Voting Counts*, 80.

6 percentage points.<sup>33</sup> This result is far from the 0.6 percentage point margin of error that a true proportional formula would have achieved in Ireland.<sup>34</sup> Furthermore, when the disproportionality index was calculated for Western countries for the 1990s, STV placed last among the proportional formulae.<sup>35</sup>

In addition to its failure to achieve proportionality, STV also falls short of attaining OPOV. Like SMP, STV relies on district vote totals to decide election results. This makes STV subject to the same problems involving OPOV faced by SMP discussed earlier.

Unlike SMP, however, one's first choice does not always count in an STV election. Instead, voters are asked to rank candidates in multi-member districts so that their second, third, etc. preferences can be considered if their first choice candidate fails to be elected or is elected with a surplus.<sup>36</sup> This causes three problems. First, it detracts from STV's ability to achieve OPOV. Voter equality is not realized if one individual is able to elect their first choice while another is only able to elect their third. Second, as mentioned above in the discussion of MMP, permitting voters to rank candidates from the same party would promote intra-party competition. Third, ranking candidates from different parties promotes the election of centrist parties. For example, consider a district contested by the Left Party, the Centre Party, and the Right Party that has weak support for both the LP and RP. If the LP has the weakest support of the three parties, its candidates would be declared defeated first, and its votes would need to be transferred to its supporters' lower preferences. Since the CP is the closest to the LP ideologically, it would be most logical for it to be the beneficiary of vote transfers from defeated LP candidates. The

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<sup>33</sup> "Irish Election 25 February 2011," last modified November 3, 2011, [http://www.tcd.ie/Political\\_Science/staff/michael\\_gallagher/Election2011.php](http://www.tcd.ie/Political_Science/staff/michael_gallagher/Election2011.php).

<sup>34</sup> *Ibid.* Since Ireland elected 165 members in 2011, the use of a proportional formula would have ensured, with minor exceptions, that the margin of error could not have exceeded  $1/165 \times 100$  percentage points.

<sup>35</sup> Pilon, *The Politics of Voting*, 57.

<sup>36</sup> Rae, *The Political Consequences of Electoral Laws*, 36-7.

same would be true if RP candidates were declared defeated. Therefore, the CP would have a systematic advantage over the LP and RP.

Although some may claim that such political convergence is beneficial, it is not what we should strive for in a democracy. If it is the desire of a portion of the electorate to be represented by a particular party—whether or not many consider this party “extreme”—it is their democratic right to obtain this representation so long as their numbers warrant at least one representative. It is categorically undemocratic to make use of an electoral formula that is structured in a manner that systematically promotes the transfer of votes from left and right wing parties to centrist parties. An electoral formula “ought not to be so structured that one specific party will be given a permanent advantage in the manner in which votes are translated into seats.”<sup>37</sup>

The use of droop quotas in STV is the source of another flaw. Droop quotas reduce the number of votes candidates require to be elected, which, in turn, wastes votes. For example, in a district with 10,000 votes cast that is to elect four representatives, the droop quota would be  $\frac{10,000}{4+1} = 2,000$ .<sup>38</sup> This permits up to 2,000 votes, or 20%, to be wasted. Although this theoretical upper limit for a four-member district is lower than the average in practice for SMP, it is still far from ideal. It is certainly possible to ensure that a much smaller fraction of the votes is wasted. Indeed, with Canada’s 308-member House of Commons and 18 parties contesting the election in 2011,<sup>39</sup> this upper limit could have been reduced to a mere 1.46%.<sup>40</sup>

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<sup>37</sup> Qualter, *The Election Process in Canada*, 83.

<sup>38</sup> Rae, *The Political Consequences of Electoral Laws*, 36.

<sup>39</sup> Elections Canada, *Forty-first General Election May 2, 2011: List of Confirmed Candidates* (Canada: Chief Electoral Officer of Canada, 2011), viii, accessed August 2, 2013, <http://www.elections.ca/ele/pas/41ge/can/loc.pdf>.

<sup>40</sup> See Appendix A for an explanation of how to calculate the theoretical upper limit to the fraction of votes that can be wasted by a proportional formula.

Finally, STV requires a significant increase in district size. According to Pilon, STV requires a minimum of five members per district to produce its semi-proportional results.<sup>41</sup> This is a significant departure from the single-member districts currently used in Canada. Not only would it decrease the level of local representation, it would also significantly increase the number of candidates contesting each district. This would make the voters' task of gathering information about all of the candidates more difficult.

#### 5.4 CONCLUDING REMARKS

Although many issues were discussed in this section, only four distinct problems were actually examined: proportionality, local representation, accountability, and party unity. The issues of wasted votes, not adhering to OPOV, strategic voting, exacerbated regional fractionalization, and gerrymandering are all symptoms of a more general problem, a lack of proportionality. As was demonstrated, if the issue of proportionality is not completely addressed, its symptoms do not disappear. Therefore, addressing this problem should receive the highest priority.

Addressing all of the problems discussed in this section will make for a difficult task but a necessary one. Given the importance of the mechanisms in place to choose those that will represent us, we should not accept formulae as flawed as SMP, STV, and MMP. Indeed, as Rae aptly notes, "*electoral laws* are of special importance for every group and individual in the society, because they help to decide who writes other laws."<sup>42</sup> Therefore, it is time to invent a new solution to the problem of choosing representatives for an elected assembly.

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<sup>41</sup> Pilon, *The Politics of Voting*, 132.

<sup>42</sup> Rae, *The Political Consequences of Electoral Laws*, 3.

## 6 DUAL-MEMBER MIXED PROPORTIONAL

### 6.1 THE BASICS

DMP is a proportional electoral formula that would require minimal changes to Canada's electoral system, aside from the electoral formula itself, to be implemented. The first necessary change would be the transformation of Canada's single-member districts into two-member districts. To maintain the size of the House of Commons, districts would be amalgamated rather than given an extra MP. This would represent a minor reduction in local representation and would not affect the average voter to representative ratio. The second required change would be less substantial. Each voter would still mark only one "X" on the ballot, but, in most circumstances, this vote would be cast for two ranked candidates running under the same party banner.<sup>43</sup> An example of this type of ballot is provided in Appendix B and is labeled Option A.

To elect two members from each district, DMP makes use of two methods. The first seat in each district is assigned by plurality (these will be referred to as the first seats). The second seat in each district, in general, is assigned in a manner that achieves proportional results at the federal level (these will be referred to as the second seats). The seat assignment process can be broken down into three straightforward steps:

1. Determine the number of seats each party deserves:
  - a. Determine the fraction of the federal vote each party has received.
  - b. Using the fraction determined in step a, calculate the number of seats each party deserves in the House of Commons:

$$\begin{aligned} & (\textit{fraction of federal vote received}) \times (\textit{total seats available}) \\ & = (\textit{\# of seats party deserves}) \end{aligned}$$

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<sup>43</sup> The two exceptions to this would be when a party nominates only one candidate and when an independent seeks election.

Record the whole number and any remaining fraction (called the remainder) of seats each party deserves.

- c. Determine the number of seats that remain to be allocated by summing the remainders of all of the parties. Allocate these seats to the parties with the largest remainders.
2. Determine the winners of the first seats using plurality.
  3. Determine the winners of the second seats:
    - a. For parties that won first seats, divide the fraction of the vote received within the districts they won first seats in half. The purpose of this step will be explained in Section 6.1.1.
    - b. For each party, create a “party list.” Place candidates that received the largest fraction of the vote in their district at the top and those that received the lowest fraction of the vote in their district at the bottom.
    - c. Determine the number of seats each party needs,  $N$ , in addition to the first seats they have been assigned.
    - d. Assign each party the first  $N$  seats from its party list.
    - e. If multiple parties have been assigned the same second seat, assign the seat to the party that performed the best in the district. Assign parties that lost a seat due to a conflict the next seat on their party list.
    - f. Repeat step e until no seat assignments conflict.

### 6.1.1 AN EXAMPLE

Consider three parties (A, B, and C) competing in an election for a ten-member assembly. Since DMP is to be used to decide the outcome of the election, the region over which the assembly governs is divided into five two-member districts. The hypothetical election data is displayed in the table below:

Party	Votes Received/Percent of Votes Received in Each District									
	District 1		District 2		District 3		District 4		District 5	
A	1504	55.7%	578	19.5%	74	20.3%	1235	29.4%	678	34.8%
B	836	30.9%	237	8.0%	193	53.0%	1468	34.9%	572	29.4%
C	362	13.4%	2145	72.5%	97	26.6%	1504	35.7%	698	35.8%

#### Step 1: Determine the Number of Seats Each Party Deserves

The following table displays the results of steps 1.a and 1.b:

Party	Total Votes Received	Percent of Votes Received	Whole Number of Seats Deserved	Remainder
A	4069	33.40%	3	0.340
B	3306	27.14%	2	0.714
C	4806	39.45%	3	0.945

To carry out step 1.c, the number of seats that remain to be allocated must be determined. This is done by adding the entries in the Remainder column, which yields two. Since parties B and C have the largest remainders, they are each allocated another seat. The final seat count for each party is given in the table below:

Party	Whole Number of Seats Deserved
A	3
B	3
C	4

### Step 2: Determine the Winners of the First Seats

To carry out step 2, simply select (for each district) the party that received the most votes:

Party	Percent of Votes Received in Each District				
	District 1	District 2	District 3	District 4	District 5
A	55.7%	19.5%	20.3%	29.4%	34.8%
B	30.9%	8.0%	53.0%	34.9%	29.4%
C	13.4%	72.5%	26.6%	35.7%	35.8%

### Step 3: Determine the Winners of the Second Seats

Step 3.a is to cut the percentage of the vote received by the winners of the first seats in half.

This produces the following results:

Table A					
Party	Percent of Votes Received in Each District				
	District 1	District 2	District 3	District 4	District 5
A	27.85%	19.5%	20.3%	29.4%	34.8%
B	30.9%	8.0%	26.5%	34.9%	29.4%
C	13.4%	36.25%	26.6%	17.85%	17.9%

In explaining DMP to others, I have found this step to cause the most confusion. As such, additional explanation is in order. When a party that has won the first seat in a district is considered for the second seat in that district, the fact that the party's votes have already been

used to elect a candidate must be accounted for by dividing the fraction of the vote received by the party in the district in half. Such a step is necessary because, should the second candidate also be elected, the party's votes would be legitimizing the election of two, not one, candidates. Therefore, this process ensures that the second candidate has sufficient support in the riding to be elected. Even though the other parties have two candidates on the ballot as well, only their first candidates can be elected at this point. Therefore, should they win the second seat, all of the party's votes in the district would be associated with one candidate and would not have contributed to electing a second candidate for the party.

Step 3.b is to create a ranked list of candidates for each party. This is done using table A. To observe how to make the lists, let us look at how to make the list for Party A. The first step is to determine where Party A performed best after the vote tallies have been adjusted for the first seat winners. In this case, Party A had its best result in district five. District five is, therefore, the first entry on Party A's list.

Party Lists		
Party A	Party B	Party C
5		

After district five, Party A performed best in district four. District four is, therefore, the second entry on Party A's list.

Party Lists		
Party A	Party B	Party C
5		
4		

This process is repeated until each party has every district it contested placed, in order of decreasing performance, on its list.

Party Lists		
Party A	Party B	Party C
5	4	2
4	1	3
1	5	5
3	3	4
2	2	1

To carry out step 3.c, the number of first seats each party won must be determined. The following table lists the number of seats each party has won so far:

Party	Seats Assigned
A	1
B	1
C	3

The number of second seats each party needs can now be calculated.

Party	Whole Number of Seats Deserved	Second Seats Needed
A	3	$3 - 1 = 2$
B	3	$3 - 1 = 2$
C	4	$4 - 3 = 1$

Step 3.d is to assign each party the number of second seats it needs from its party list. To accomplish this, we first attempt to assign the parties' seats to their best performing candidates:

Party Lists		
Party A	Party B	Party C
5	4	2
4	1	3
1	5	5
3	3	4
2	2	1

Following this, step 3.e is to look for and address conflicting seat assignments. In this case, the second seat in district four has been assigned to both Party A and Party B. To address the conflict, the results in Table A are observed. Party B outperformed Party A in district four; therefore, Party A must attempt to take the second seat in the next district on its party list, district one:

Party Lists		
Party A	Party B	Party C
5	4	2
<del>4</del>	1	3
1	5	5
3	3	4
2	2	1

Step 3.f is to resolve any conflicts that were produced as a result of step 3.e. This is accomplished by repeating step 3.e. The result is the following:

Party Lists		
Party A	Party B	Party C
5	4	2
<del>4</del>	1	3
<del>1</del>	5	5
3	3	4
2	2	1

The seat assignments are now free of conflicts, and all second seats have been filled. The tables below summarize the election outcome:

Party	Total Votes Received	Percent of Popular Vote Received	Whole Number of Seats Deserved	Remainder	Number of Seats Won
A	4069	33.40%	3	0.340	3
B	3306	27.14%	2	0.714	3
C	4806	39.45%	3	0.945	4

District #	Seat 1	Seat 2	Place
1	A	B	2
2	C	C	2
3	B	A	4
4	C	B	2
5	C	A	2

The fourth column in the second table indicates the place of the candidate that won the second seat. This is not done for the first seats since the winner always places first.

### 6.1.2 NOTES ON EXAMPLE

At first glance, it may seem odd that Party A placed fourth in district three when there were only three parties competing, but there were actually four candidates competing for the seats in the district. In district three, the fourth candidate to enter the competition was the second candidate on Party B's ballot. Since the second candidate on a party's ballot can only have a chance of winning if the first candidate on the ballot wins the first seat in the district, the second candidates of the parties that do not win the first seat are never considered. Therefore, the one party that will have an opportunity to elect both of its candidates contributes an extra competitor to the local race.<sup>44</sup>

There are a few properties of this election outcome that are worth noting. First, four of the five districts, or 80%, are represented by two parties. Second, only one seat (the second seat in district three), or 10% of the total seats, was assigned differently than it would have been under Two Member Plurality (TMP). Finally, the results are as proportional as possible with ten seats. As will be seen later, these results do not seem to be special. Rather, they appear to be normal occurrences when DMP is used.

Finally, it is important to point out that the results calculated in the previous section can be arrived at in seconds using a computer, even when calculating results for a federal election. In Section 9, results calculated using a computer program that implements DMP will be presented for past federal elections.

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<sup>44</sup> This is true so long as an independent does not win the first seat. Since votes for independents cannot be transferred to other candidates, the number of candidates competing for the seats in a district when an independent wins the first seat is: (number of parties) + (number of independents). If a party candidate wins the first seat in a district and this candidate's party nominated two candidates, then the number of candidates competing in the district is: (number of parties) + (number of independents) + 1.

### 6.1.3 GENERAL NOTES ON THE DESIGN OF DMP

DMP is designed to produce election outcomes as similar to those produced by TMP while achieving proportionality. Obviously, DMP must fail to achieve equality with the results obtained when using TMP in order to produce proportional election outcomes. Therefore, the objective was to minimize, not eliminate this difference. These two disparate objectives have been accomplished with three techniques. First, proportionality is achieved by using the federal vote totals to allocate seats to each party. Second, the formula strives to assign parties their deserved number of second seats where they have had their strongest performances. Third, the second place finisher in each district is elected as frequently as possible. The last two mechanisms that have been built into DMP work to push the correction factor to as small a value as possible. As will be demonstrated in section 9.2.1, the correction factor tends to be surprisingly small.

It is also worth mentioning that DMP does not make use of the traditional methods to allocate seats to parties, such as the d'Hondt Highest Average formula.<sup>45</sup> Many of the traditional methods do not achieve strict proportionality and were likely used to make calculating election results easier. This computational efficiency, however, has been made unnecessary by modern computers. Therefore, DMP has been designed to directly calculate the number of seats each party deserves by simply multiplying parties' fractions of the popular vote by the total number of seats in the elected assembly. Although this is identical in principle to the Largest Remainder formula, it omits the roundabout calculations that were likely introduced for computational efficiency.<sup>46</sup> Thus, by allocating seats to parties in this manner, strict proportionality is achieved in the most straightforward manner.

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<sup>45</sup> For an explanation of how the d'Hondt Highest Average formula works, see Rae 1971: 31-34.

<sup>46</sup> For an explanation of how the Largest Remainder formula works, see Rae 1971: 34-35.

## 6.2 BALLOT DESIGN

In addition to the status quo ballot design mentioned in Section 6.1, I am also putting forward the ballot labelled as Option B in Appendix B. This design provides the electorate more flexibility than Option A by allowing them to split their vote into ten equally valued portions. Such flexibility would allow voters to make more precise and nuanced decisions. Instead of deciding which party one wants to win all of the seats in the elected assembly, voters would be able to choose their ideal seat distribution between the parties. This would lead to three benefits. First, voters would be able to decide if they wanted a majority or a minority government. For example, if a voter wanted a majority government, they could give six of their votes to their preferred party and their remaining four votes to the party or parties that they want to form the opposition. This leads to the next benefit of this method of voting: not only does the voter get to choose their government, they also get to have a say in who will form the opposition. Finally, voters could give support to single-issue parties such that they cannot form the government. For example, consider a voter that generally supports Party A but is concerned about an issue, call it Z, that Party A is not addressing. If a party were to be formed solely for the purpose of addressing issue Z, call them Party B, this voter may wish to give them support while still wanting Party A to form the government. Under this ballot design, such a voter could give a minority of their vote to Party B and a majority to Party A. This would put pressure on Party A to address issue Z without risking the possibility of Party B forming the government.

At this point, some may be wondering why I have not also suggested a ballot design option that would allow the electorate to cast separate votes for parties and candidates. I will address this potential concern now. As a consequence of DMP's structure, allowing the electorate to vote for Party A with their party vote and Candidate C from party B with their candidate vote

would lead to contradictory results. Voting in this manner would result in parties A and B both placing first in the district. Consequently, one party would have to be arbitrarily chosen as the true winner. In the case of DMP, Party A would have to be given this label. However, this would cause two problems. First, Candidate C would win the first seat despite their party not receiving any of the party votes. This would significantly reduce DMP's ability to produce proportional results, since Party A would deserve to win both seats in the district but would only be able to win the second. Second, Party A's candidates could place fifth, for example, in the district, which would significantly increase the magnitude of the correction factor. Therefore, in light of these problems, I do not recommend allowing the electorate to split their vote in such a manner.

### **6.3 THE EXCEPTION: INDEPENDENTS**

In general, DMP assigns the first seats using plurality and the second seats with a proportional algorithm. However, in order to prevent independents from being disadvantaged, an override mechanism has been built into DMP. In cases where an independent places second in a district, the proportional algorithm used to assign the second seats is suspended, and both seats in the district are assigned by plurality. This is the only circumstance where DMP does not assign second seats on a proportional basis.

### **6.4 THE THRESHOLD**

Every electoral formula has a minimum number of votes that a candidate or party must acquire to achieve representation, or a threshold. SMP has a variable threshold. It varies from district to district and from one election to the next. It depends on the number of candidates

contesting a district's seat and the manner in which the votes are distributed between the candidates. However, a government-mandated threshold is not imposed, at least not in Canada. Consequently, it is possible to win with an arbitrarily small percentage of the vote. The more candidates that contest a district's seat, the smaller the minimum fraction of the vote a candidate will require to win the seat will be. For example, if there are twenty candidates contesting a district, the seat could be won with as little as 5% plus one of the votes.

Although it is commonly accepted to not mandate a threshold when using SMP, it is unusual to see a proportional formula in use without one. However, thresholds are arbitrary and, if high enough, can pose a significant barrier to new parties. Therefore, the task of choosing a threshold should not be seen as trivial. Moreover, we should be inclined to support low thresholds and demand rigorous, democratic justifications for high thresholds. With this in mind, I would propose using a 2% threshold if DMP were to be used. I am recommending this threshold for four reasons. First, it would have been high enough to block all but the top five parties from winning seats in the past four federal elections. Second, it is high enough that a party would require more than a few protest votes to win representation. In other words, a party would require significant public support to gain seats in the House of Commons. Third, it is low enough that it would not present a significant barrier to new parties gaining representation. And, finally, it is a moderate choice, as it is in the middle of the range of thresholds that are already in use.<sup>47</sup>

It should be noted that this threshold would only apply to the second seats. Since the first seats would be assigned by plurality, a party with fewer votes than the threshold could still win seats. However, this would be a rare occurrence.

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<sup>47</sup> Thresholds appear to range from 0.67% to 5% (Law Commission of Canada 2004, 91; Seidle 1989, 255).

## 6.5 ADDRESSING TIED VOTES

With SMP, there is only one opportunity for a tie: two or more candidates can tie for first place in a district. To address this issue, section 318 of the Canada Elections Act requires that a by-election be held for the districts in which a tie for first place has occurred.<sup>48</sup> Not only is this a costly solution to the problem, it also has no democratic merit. Furthermore, it provides the voters in such a district with an unfair advantage over other voters. On election night, no one knows what the result of the election will be before the votes are counted. That the voters' choice in a district with a tie for first place has been indecisive should not give them the privilege of knowing the election outcome in every other district before they choose their representative. Therefore, another solution to this problem must be devised.

Democratic theory does not handle ties well. Consequently, there is no obvious solution to deal with this problem. The current solution (to hold a by-election) is costly, time consuming, and unconvincing. Some formulae make the decision of how to deal with ties easier by restricting our choice of remedy. DMP is such a formula. Even if one thought the current manner in which ties are dealt with was appropriate, it could not be used to address the issue when using DMP. This is evident if we consider the points at which DMP permits ties to occur. When DMP is used, ties can occur when:

1. Deciding candidate placements in the districts, which has two instances that can be of consequence:
  - a. Deciding the winner of the first seat
  - b. A conflicting seat assignment is addressed
2. Ordering the party lists

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<sup>48</sup> "Canada Elections Act: Part 15," last modified June 22, 2012, <http://www.elections.ca/content.aspx?section=res&dir=loi/fel/cea&document=part15&lang=e#sec318>.

### 3. The remainder seats are allocated

Holding a by-election could not address the ties that occur at points two and three. Whom would the by-election be held between for the second type of tie, and where would it be held? Would one completely redo the election to break the third type of tie? By-elections are completely nonsensical in these situations.

The first type of tie would permit a by-election solution; however, since the result obtained when this tie is broken has the potential to affect the results in other districts, one would have to wait until the by-election was held in order to calculate the results of the election in every other district. Not only would this likely be met with negative reaction from the public, in the case of tie 1.a, it may end up being of no consequence at all. Since it can be expected that between 87 and 89% of the seats will be assigned to the first and second place finishers, the two individuals that tied for first (it would be extremely unlikely to have a three, four, etc. way tie) would likely both win a seat in the district anyway.<sup>49</sup> Finally, there is a more cost-effective and timely solution. For these reasons, I do not support holding a by-election to break the first type of tie.

To address all three types of ties, DMP chooses a winner by random draw. This may seem odd or illegitimate, but there really is no legitimate solution. The voters have chosen not to decide, and democracy only functions by having a decisive vote. When it fails to produce one, a decisive answer must be manufactured. Whether it is manufactured through a by-election or a random draw, it is manufactured nonetheless. Although this solution is as unconvincing as holding a by-election, it would cost nothing and would consume no extra time.

In order to ensure that the implementation of the DMP algorithm that I have written produces the same results no matter when the election data is put through the program, I have removed the tie breaking procedure from it. Instead of writing the program to randomly choose

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<sup>49</sup> This result is presented in Section 9.2.1.

a winner when a tie occurs, the program assumes that the data has been randomized in two ways before it is uploaded:

1. The order in which the parties appear on the ballot is randomly generated for each district.<sup>50</sup>
2. The order in which the party labels appear in the list of parties contesting the election is randomly generated.

To see how randomizing the election data in this way addresses the three types of ties listed above, it is useful to understand how my implementation of DMP is structured to break these ties. The three types of ties are addressed as follows:

1. The candidate closest to the top of the ballot wins.
2. The candidate that appears first in the election data file at the time of the comparison wins. Note, however, that there is no structural benefit given to candidates based on the order of the election data. Since the process of ordering the party lists can shift tied candidates' positions before they are compared, it cannot be said that candidates closer to the top are in a better position than those at the bottom or vice versa.
3. The party closest to the top of the list of parties contesting the election wins.

Finally, it is important to note that, despite the fact that there are more ways ties can occur with DMP than SMP, they would still be a rare event. Furthermore, many of them would be of no consequence. Except for tie 1.b, each of the ties is unlikely to affect an election outcome. The reason this is true for tie 1.a was explained earlier. For the second type of tie to affect the outcome, it would have to occur at the boundary of a party list. To make this clearer, consider the example discussed in Section 6.1.1. Party B was able to elect the first two candidates on its

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<sup>50</sup> Note that this does not require the ballots used by voters to have the same ordering. The order of the ballot being referred to here is only required for use in the program. In order to prevent a systematic advantage to those closest to the top of the ballot, the order of the ballots used for voting should be changed frequently within each district.

party list. Had the candidates in districts one and four received the same percentage of their respective district's votes, it would not matter which candidate is chosen as the winner of the tie. Both would win their seats regardless of the outcome. However, if the tie were between the candidates in districts one and five, the loser of the random draw would possibly not win a seat. In this example, that is not the case. Party A performed better in district five than Party B, and, therefore, Party B's candidate in district one would win a seat even if they lost the random draw to the candidate in district five. Similarly, the third type of tie would only be of consequence if two parties tied for the last remainder seat. If two parties tied for the second last remainder seat, both parties would be allocated a seat.

## **6.6 BY-ELECTIONS**

DMP uses federal voting results in combination with local results to determine election outcomes at a local level. In a by-election scenario, there may be one seat to be filled or several, with some districts requiring both of its two seats to be filled and some only one. These circumstances would not permit the application of DMP. As a result, I am recommending that a simple plurality formula be used to decide the outcome of by-elections.

There are two reasons why this recommendation is not problematic. First, there would be no incentive to resign one's seat to hold a by-election. For example, if the Green Party won a second seat in third place in a district where the Conservative Party placed strongly in second, the party that stands to gain from having a by-election in the district, the Conservative Party, has no control over whether the Green Party member resigns their seat. Moreover, the Green Party member would have a strong incentive to keep their seat, as they would know that their party would likely lose the by-election. Second, it is not much of a concern if the outcomes of by-

elections are not proportional. In the 16 years between 1998 and 2013, there has been an average of only 12 by-elections every four years, and the most districts that have had a by-election at once is seven.<sup>51</sup>

## **6.7 WHEN DMP FAILS TO PRODUCE PROPORTIONAL RESULTS**

### **6.7.1 CASE 1: ONE OR MORE PARTIES WINS A FRACTION OF THE FIRST SEATS GREATER THAN TWICE THEIR FRACTION OF THE POPULAR VOTE**

Under extreme circumstances, DMP would fail to produce proportional results. One of the two situations where such a failure would occur is when a party is able to secure a fraction of the first seats greater than twice its fraction of the vote. This would cause such a party to be over-represented and the remaining parties to be under-represented. So that no one party is penalized more than another, DMP has been constructed to distribute the impact of parties that have been over-represented proportionally across all remaining parties. For example, if party A obtains 40% of the vote but wins 100% of the first seats, they will have secured 50% of all of the seats before the second seats are assigned. As a result, there would be no way to assign the second seats to produce a proportional outcome. Party A would be over-represented by 10 percentage points, and the remaining parties would be collectively under represented by the same amount. If there were two other parties, B and C, that received 25% and 35% of the vote respectively, party B would be entitled to  $\frac{25\%}{60\%} \times 50\% = 20.8\%$  of the seats and party C would be entitled to

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<sup>51</sup> “Past Elections,” last modified June 17, 2013, <http://www.elections.ca/content.aspx?section=ele&dir=pas&document=index&lang=e>.

$\frac{35\%}{60\%} \times 50\% = 29.2\%$  of the seats.<sup>52</sup> Therefore, each party would lose the same fraction of its deserved seat share, 17%.

There are two important points that must be noted. First, such a situation has never arisen at the federal level in Canada's history.<sup>53</sup> Therefore, it seems unlikely that such a failure would be experienced if DMP were to be implemented. Second, and most importantly, DMP guarantees that any party which receives less than or equal to 50% of the federal vote will be denied a majority government.<sup>54</sup> Therefore, DMP prevents the most harmful consequence of over-representation: false majority government.

#### **6.7.2 CASE 2: ONE OR MORE PARTIES WINS FEWER SEATS THAN THEY DESERVE BECAUSE THEY DID NOT RUN A FULL SLATE OF CANDIDATES**

DMP attempts to assign parties second seats where they performed the best. Since this cannot always be accomplished, parties may be assigned some seats close to or at the bottom of their party lists. When a party does not run a candidate in every district or, in some cases, two candidates, it runs the risk of not winning its deserved number of seats.<sup>55</sup> If a party must be assigned second seats in districts where it does not have candidates that could win the second seats, then such a party will lose some of its deserved seats. These lost seats are redistributed

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<sup>52</sup> To distribute the impact of party A's over-representation proportionally across parties B and C, the total number of seats is taken to be 50% of the original, and the total number of votes is taken to be 60% of the original. The number of seats parties B and C deserve is then calculated using the same formula given in section 6.1, step 1.b.

<sup>53</sup> "Canadian Election Results by Party: 1867 to 2008"; "Electoral Results by Party."

<sup>54</sup> There are two caveats to this statement. First, if votes are cast for parties or independents that fail to achieve their deserved representation, the other parties will receive more than their deserved representation, which could transform a minority of votes into a legislative majority. Note that a party would need to be very close to having 50% of the vote for this to happen. Second, it is possible to conceive of situations where a party that is over-represented forms a coalition with one or more parties to form a majority government with a combined popular vote of less than 50%.

<sup>55</sup> If a party runs one candidate in a district where it wins the first seat, it will not have a candidate that could win the second seat. It is an absence of a candidate that can win a second seat that causes the problem outlined in this section.

between the other parties that have passed the threshold. This effect occurred in the simulation of the 2008 election, as the Green Party deserved 7% of the seats but was only able to be assigned 4%.<sup>56</sup> It is important to stress, however, that this less than desirable result arises because of a party's limited capacity to find candidates and/or a party's decision to only field candidates in certain regions of the country. Such results are not a product of a structural failure of DMP.

## 7 CONSTITUTIONAL ADHERENCE

Sections 51(1) rule 2 and 51A of the Constitution outline minimum seat allocations for each province, and Section 51(2) states that each of the three territories is entitled to one seat.<sup>57</sup> As discussed earlier, these three sections present problems for most electoral formulae. DMP, however, can meet the minimum seat allocation requirements mandated by the Constitution while still being applied at the federal level. Indeed, one of the motivations for the creation of DMP was to accomplish this goal. As a result, no matter how disproportionately the seats are distributed between the provinces or how large the differences in voter turnout are from one district to the next, DMP would achieve OPOV. Furthermore, this means that DMP would produce election results that are as proportionate as mathematically possible.<sup>58</sup>

In order for DMP to be implemented, minor constitutional change would be required. The use of two-member districts would make it necessary for each province and territory to be allocated an even number of seats. To ensure that the provinces and territories each receive an even number of seats, Section 51(1) rules 1 and 6.b would need to be amended so that fractional

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<sup>56</sup> See Appendix E for this result.

<sup>57</sup> "Constitution Act, 1867."

<sup>58</sup> The previous two statements would not be true in the exceptional cases described in Section 6.7.

remainders are rounded up to the nearest even integer.<sup>59</sup> In addition, Section 51(2) would need to assign two seats to each territory instead of one.<sup>60</sup> Since neither of these changes breaks with the “principle of proportionate representation,” the Parliament of Canada should be able to unilaterally make the adjustments.<sup>61</sup> These changes would result in Ontario, Nova Scotia, Newfoundland and Labrador, and the three territories receiving one extra seat based on the new seat allocation calculated on December 16, 2011.<sup>62</sup> It is important to note that this is not an entirely new suggestion in the electoral reform debate. For example, the Law Commission of Canada has recommended adding a seat to each territory to implement MMP.<sup>63</sup>

## 8 GERRYMANDERING

Although gerrymandering is still possible with DMP, it is made prohibitively difficult. Moreover, even if it were to be achieved, it would be almost entirely inconsequential. Both points are a result of the fact that 50% of the seats are assigned using plurality, while the remaining 50% are assigned in a manner that aims to produce proportional results. The first consequence of this is that gerrymandering could not increase the seat count of a party that acquires 50% or more of the popular vote, because the only seats that can be won through gerrymandering are those assigned using plurality. Furthermore, a party with less than 50% of the popular vote that successfully gerrymanders<sup>64</sup> would be unable to secure a majority of seats, making the few seats that they would gain over what they deserve based on proportionality of no more consequence than to deny all other parties a small fraction of their deserved representation.

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<sup>59</sup> “Constitution Act, 1867.”

<sup>60</sup> *Ibid.*

<sup>61</sup> “Constitution Act, 1982,” last modified August 30, 2013, <http://lois.justice.gc.ca/eng/Const/page-16.html#docCont>. See sections 42.1.a and 44.

<sup>62</sup> “House of Commons Seat Allocation by Province.”

<sup>63</sup> Law Commission of Canada, *Voting Counts*, 104.

<sup>64</sup> I am considering gerrymandering successful when a party is able to secure more seats than it deserves based on proportionality.

This small gain in seat count for a party that successfully gerrymanders could come about only after overt and widespread manipulation of electoral boundaries. This is again due to the reduction of seats assigned using plurality to 50% of the total. Consider a party, P, that anticipates receiving 35% of the popular vote in the next election, is part of a coalition government that does not mind participating in gerrymandering that will only serve to help party P and harm the others, and that will not be punished by the electorate for gerrymandering. In order for party P to successfully gerrymander, it must secure 70% + 1 of the first seats. If this is not yet convincing that gerrymandering with DMP is an extraordinarily difficult task, then consider that in Canada's entire history, despite the many bizarre and unfair election results our plurality formula has produced, it has never given a party a percentage of seats greater than, nor equal to, double its percentage of the vote.<sup>65</sup>

## 9 TESTING DMP IN MATLAB

### 9.1 INTRODUCTION

In order to apply DMP to past elections, it had to be assumed that each one-member district was actually a two-member district. However, if DMP were implemented in Canada without a simultaneous decision to increase the number of seats in the House of Commons, the number of districts would be reduced by approximately half. Thus, results that have been calculated for past elections using DMP cannot be taken as the exact results had it been in use at the time of the elections. There are two primary reasons for this. First, changing the district boundaries would affect where parties win their seats. Since where parties perform well enough to ultimately win their seats is not relevant to any of the goals of this project, this change is inconsequential. Second, reducing the number of seats by half would affect the formula's ability to achieve

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<sup>65</sup> "Canadian Election Results by Party: 1867 to 2008"; "Electoral Results by Party."

proportional results. This effect, although important to mention, would not be significant. This is demonstrated by the change in the formula's allowed margin of error, which is related to the number of seats in the House of Commons. For the past elections examined in this report, the maximum margin of error is 0.16 percentage points.<sup>66</sup> This would jump to approximately 0.32 percentage points if DMP were actually implemented, which is still negligible.

It should also be noted that there is much that has not been included in this report. The MATLAB files that execute the DMP formula and many of the results that it produces, such as the province-by-province results and the district-by-district results, do not appear in their entirety or at all in this report. If any reader is interested in seeing this material, they should contact the author.

## **9.2 PAST FEDERAL ELECTION OUTCOMES WITH DMP**

### **9.2.1 OVERVIEW OF RESULTS**

Appendices C through F display results for the past four federal elections. Each appendix contains seven charts. The first chart displays the distribution of the popular vote between the major parties, where major party has been defined to be any party that received 3% or more of the popular vote. The second chart displays the election outcome when DMP is applied to the election data,<sup>67</sup> while charts three through five display the correction factor and the manner in which the parties' seats are distributed across the country, including the district representation

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<sup>66</sup> Under normal circumstances, excluding the usually negligible effect of votes received by independents and parties that do not pass the threshold, each party can be under or over represented by at most one seat. Therefore, the margin of error is, at most:  $[ 1/(\text{Seats in the House of Commons}) ] \times 100$  percentage points

<sup>67</sup> A threshold of 2% was used to calculate these results. The justification for this threshold was presented in Section 6.3.

factor.<sup>68</sup> The last two charts show the results of the election under two different implementations of plurality.

Four important observations can be made with these results:

1. In each election, DMP produced proportional results, with the exception of 2008. This minor exception was explained in Section 6.7.2.
2. The correction factor ranged from 11 to 13%.
3. 78 to 81% of the districts were represented by two different parties in the aftermath of each election.<sup>69</sup>
4. The district representation factor ranged from 1.78 to 1.81.

### 9.2.2 REGIONAL REPRESENTATION

Despite its focus on creating proportional election outcomes on a federal level, the results of applying DMP to the past four federal elections in Canada demonstrate that it also achieves more proportional results than SMP on the provincial scale. Appendices E.2 and F.2 display the vote distributions, seat distributions when DMP is used, and the seat distributions when SMP is used for Quebec, Ontario, Saskatchewan, and Alberta for the past two federal elections. Alberta and Saskatchewan were chosen for their display of some of the worst distorting effects of SMP. Quebec was included to show that regional parties with concentrated support would no longer be benefitted if DMP were implemented. Finally, Ontario was included because it was one of the most accurately represented provinces.

In each province, and in each election, DMP produced more proportional results than SMP.

Although the results in Alberta, for example, were still distorted when DMP was used to

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<sup>68</sup> For an explanation of how to calculate the district representation factor, see Appendix G.

<sup>69</sup> Districts represented by a party and an independent, in addition to districts represented by two independents, are considered to be represented by two different parties.

determine the election outcomes, significant improvements were made. The ten percentage point reduction in the Conservative Party's representation in both elections allowed the other major parties to collectively increase their representation in the province by 3.5 times. Furthermore, this reduction in the Conservative Party's representation allowed DMP to accurately reflect the political diversity of Alberta, since every major party that received support in the province was able to be awarded representation; thus, increasing the number of parties represented in Alberta from two to four.

The trends displayed in Appendices E.2 and F.2 can be observed in the provincial results not included in this report, as well as in the 2004 and 2006 elections. This is significant because it means that DMP could reduce the regional fractionalization that is routinely manufactured by SMP when slight concentrations of support occur. Political parties would have more regionally diverse caucuses, reducing the regional biases that often exist within parties. In addition, the incentive to participate in regionally inflammatory rhetoric would no longer exist since DMP would not reward any regional concentration of support for regional or federal parties.<sup>70</sup> Finally, since regional parties have relied on the distorting effects of SMP for their success and would not experience such inflated representation with DMP, the implementation of DMP would significantly diminish the strength and, therefore, relevance of regional parties.

A related issue is the scope of party representation across the country. That the district representation factor ranged from 1.78 to 1.81 in the elections examined is highly significant. This means that each party would have been represented in nearly twice as many districts as it was in our single-member district system, which further supports the idea that parties would have

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<sup>70</sup> Although DMP may over-represent a federal party, such as the Conservative Party in Alberta, in one or more regions, it will not over-represent them in the elected assembly. In other words, a party receiving more seats than it deserves in a region will not contribute to that party receiving more than its deserved seats in the elected assembly.

had more regionally diverse caucuses. More importantly, however, governments and oppositions would have been more regionally diverse.

Table B below displays the breadth of government and opposition representation for the past four federal elections when DMP is used to calculate the outcomes. For each election, a left-centre and right-centre coalition comprised of the minimum number of parties to form a majority government is examined. These coalition governments would have been represented in 78-88% of the districts, with an average district representation of 84%. Impressively, all of the oppositions to these coalition governments would have been represented in a majority of districts. They would have ranged from 52-82% district representation, with an average district representation of 67%. To put this in other terms, Canadians would have had an 84% chance of being represented by a member of the governing coalition and a 67% chance of being represented by a member of the opposition, on average.

<b>Table B</b>			
Year of Federal Election	Government/Opposition	Percent of Seats	Percent of Districts
2004	Government A: Liberal, NDP	53%	80%
	Opposition A	47%	74%
	Government B: Liberal, Conservative	67%	88%
	Opposition B	33%	54%
2006	Government A: Liberal, NDP, Green	53%	83%
	Opposition A	47%	78%
	Government B: Liberal, Conservative	67%	86%
	Opposition B	33%	52%
2008	Government A: Liberal, Conservative	66%	84%
	Opposition A	34%	52%
	Government B: Liberal, NDP, Green	50.3% <sup>71</sup>	83%
	Opposition B	49.7%	82%
2011	Government A: Liberal, NDP, Green	54%	88%
	Opposition A	46%	80%
	Government B: Liberal, Conservative	59%	78%
	Opposition B	41%	60%

<sup>71</sup> The government would have had 310 of 616 seats in this case.

The best result that has been realized in Canada at the federal level in terms of district representation for a government occurred in 1958 when the Progressive Conservatives won 78.5% of the seats and were, therefore, represented in the same fraction of the districts.<sup>72</sup> Since each district only has one member in an SMP system, it is impossible for a party to be represented in a larger fraction of the districts than its fraction of seats in the elected assembly. Therefore, the opposition had to be represented in exactly 21.5% of the districts. In contrast, the average district representation for the eight coalition governments examined above is six percentage points higher than the best result that has ever been achieved with SMP at the federal level. Moreover, the average district representation for the opposition is greater than four times what would be possible in an SMP system that produced a government represented in 84% of the districts.

## 10 THE CORRECTION FACTOR AS A STRENGTH OF DMP

As noted in Section 9.2.1, the correction factor ranged from 11 to 13% when DMP was used to calculate outcomes for the past four federal elections. Aside from the fact that this is a small fraction of the seats, it must be remembered that it is the 11-13% of the seats that were assigned differently than they would be under TMP that produced the proportionate results. Therefore, although a candidate may win a seat despite placing fifth in their district, they deserve to win based on the federal support for their party. What is surprising about DMP is not that these seats exist but that their number is so small—DMP is able to correct the distortions produced by plurality with a very small correction factor.

Since DMP is a proportional formula, parties would have an incentive to campaign in areas where they do not expect to win if it was implemented. However, since parties could also win a

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<sup>72</sup> “Canadian Election Results by Party: 1867 to 2008”; “Electoral Results by Party.”

seat in an area where they do not expect to do well (i.e., place first or second), there would be an incentive not only to campaign in all districts but to run quality candidates in all districts as well. A party would not want its caucus to be filled with incompetent individuals, and neither would its supporters. As a result, the practice of nominating ‘ballot fillers’ in districts where a party performs poorly would likely be less common, if not ended entirely. It would also raise the bar on the quality of electoral competition, since strong performing incumbents would be up against quality candidates from the other parties.

At first glance, this may seem like it would make a small party’s work more difficult, since under the current system it can focus on nominating quality candidates in the few areas where it performs well. However, if DMP were to be implemented, there would likely be 172 districts instead of 338.<sup>73</sup> Therefore, small parties, such as the Green Party, that have not done well enough to win both seats in many districts would only need to focus on nominating one quality candidate in most districts. The only districts where two quality candidates should be nominated are those where the party has a chance of winning the first seat. As a result, small parties would only need to nominate approximately half as many candidates as they would with SMP to contest every district.

Finally, voters would still be able to hold candidates accountable. If a candidate were to win a second seat in 5<sup>th</sup> place, it would be very unlikely that the results of the next election would align in such a way to elect the candidate in 5<sup>th</sup> place again. Therefore, a candidate elected in such a position would have until the next election to prove to the voters of the district that they were worthy of election. If they fail to do so and again come in 5<sup>th</sup> place, they would most probably not be re-elected. The 2008 and 2011 federal elections add validity to this claim.

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<sup>73</sup> As noted in Section 7, six extra seats would be required to implement DMP. Therefore, there would be 344 seats in the House of Commons instead of 338.

In 2008, the Green Party received 3% of the vote in Quebec but 11% of the seats with DMP, whereas, in 2011, the party received 2% of the vote in Quebec but 3% of the seats.<sup>74</sup> This demonstrates that a party rejected by voters twice in a row is unlikely to retain its seats. What is also promising about this result is that not one seat that the Green Party won in 2008 in Quebec with DMP was retained in the 2011 election.<sup>75</sup>

## 11 CHALLENGING ACCEPTED LIMITATIONS OF ELECTORAL FORMULAE: WHY DMP COULD BRIDGE THE GAP IN THE ELECTORAL REFORM DEBATE

The unique property of DMP is the mechanism used to decide the winners of the second seats. Using the federal voting results to determine the number of seats each party deserves and the individual district results to decide where each party will win its seats breaks sharply with the accepted structures of electoral formulae. It is explicitly<sup>76</sup> or tacitly<sup>77</sup> accepted by most actively involved in the electoral reform debate that the following are mutually exclusive: small districts and proportional results. If one wants to achieve proportional results, reformers often present two general options:

1. Create large multi-member districts where the winners are decided solely by the votes cast within the district.
2. Retain single-member districts, but achieve proportional results by adding a second tier of representatives that will be elected based on a regional or federal vote tally.

To implement option two, one must decide whether to increase the size of the legislative body or decrease the number of single-member districts. This option does not break with the mutually

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<sup>74</sup> See Appendices E.2 and F.2 for these results.

<sup>75</sup> In 2008, the Green Party placed sixth or seventh in each Quebec district that they won a seat. Furthermore, the Green Party did not markedly improve on these results in the 2011 election and, in most cases, lost support.

<sup>76</sup> Rae, *The Political Consequences of Electoral Laws*.

<sup>77</sup> Law Commission of Canada, *Voting Counts*; Pilon, *The Politics of Voting*.

exclusive relationship noted above because it achieves proportionality by superimposing large multi-member districts over smaller single-member districts.

This leads Rae to present the following dilemma:

Suppose that the reformer seeks to optimize three values: (1) the compactness of electoral districts, (2) the smallness of the legislature elected from these districts, and (3) the proportionality of the election outcomes. Increasing district magnitudes will force the reformer to sacrifice one of his first two values, because he must either (a) make the districts less compact to include more seats, or (b) leave the district compact, but add seats, thereby making the elected legislature a larger body.<sup>78</sup>

This issue has plagued much of the reform debate. Those advocating for reform often disagree on the values that should be retained and those that can be compromised if necessary. This disagreement can be seen between those that support STV, who sacrifice single-member districts for semi-proportional results, and those that support MMP, who sacrifice either the small size of the legislative body or the number and small size of the single-member districts.

DMP implements the only algorithm for allocating seats that allows all of the values presented by Rae to be satisfied.<sup>79</sup> It allows the size of the elected assembly to remain small, produces results that are generally as proportionate as mathematically possible, and maintains a high degree of district compactness. The ability of DMP to adhere to these basic values and those outlined in Section 4 could allow the formula to unite the supporters of STV and MMP.

## 12 CONCLUSION

The electoral formula reform debate in Canada has stalled. This report has put forward a new solution in the hope of re-igniting the debate. Unlike the leading alternatives to SMP, DMP can meet the criteria detailed in Section 4 with minor exceptions. It would allow local

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<sup>78</sup> Rae, *The Political Consequences of Electoral Laws*, 119.

<sup>79</sup> I should add a word of caution here. There are likely many algorithms for assigning seats that I have never heard of, and, as a result, there is the possibility that another algorithm exists that satisfies these criteria. However, if it does exist, it is not being discussed. Therefore, as far as the electoral reform debate goes, it might as well not exist.

representation and the size of the House of Commons to be retained while achieving proportional results with almost flawless accuracy. Furthermore, the evidence presented in Section 9 suggests that DMP could produce other benefits that go beyond what is necessary. Not only does it seem to substantially increase the district representation of governments and oppositions beyond their fraction of the seats, the evidence also suggests that DMP would produce more proportional results at the provincial level than SMP. Surprisingly, DMP is able to accomplish all of this with only a small deviation from the results produced by TMP.

#### AFTERWORD

Months after developing DMP, I came across a recently developed and sparsely discussed electoral formula called Fair Majority Voting. The main algorithm used by this formula is similar to the algorithm I developed to assign the second seats in DMP.<sup>80</sup> The main difference between the two formulae is that DMP makes use of two-member districts, whereas Fair Majority Voting uses single-member districts.<sup>81</sup> This has been brought to the reader's attention for the sake of full transparency.

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<sup>80</sup> Michel Balinski, "Fair Majority Voting (or How to Eliminate Gerrymandering)," *The Mathematical Association of America* 115 (February 2008): 99-101, <http://www.mathaware.org/mam/08/EliminateGerrymandering.pdf>.

<sup>81</sup> *Ibid.*, 99.

APPENDIX A: CALCULATING THE MAXIMUM POTENTIAL FOR WASTED VOTES  
WITH PROPORTIONAL FORMULAE

The upper limit to the fraction of votes that can be wasted,  $W_M$ , represents the maximum fraction of the votes that cannot count to the election of any representatives. It is important to realize that this quantity does not tell us the fraction of votes that *will* be wasted but, rather, the highest fraction of votes that *could* be wasted. This quantity is the smallest when a proportional formula is used. The purpose of this appendix is to derive the formulae required to calculate  $W_M$ . For this derivation,  $p$  will be used to denote the number of parties and  $s$  the number of seats. Furthermore, the specific case when  $s$  is 5 will be used to develop the formulae for the general case.

Two separate cases will need to be considered. The first case will deal with situations where the number of parties is less than twice the number of seats. In order to derive the formulae for this case, two sub-cases will need to be examined: when the number of parties is even and when the number of parties is odd. The second case will handle when the number of parties is greater than twice the number of seats.

Two assumptions will be made in developing these formulae. First, it will be assumed that no votes are cast for independents. Since independents do not usually perform well in Canada, the effect of this omission should not be significant. Second, it will be assumed that a completely proportional electoral formula is being used.<sup>82</sup>

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<sup>82</sup> This implies that no formal threshold exists and that there is effectively only one district used to allocate seats on a proportional basis. To apply the formulae derived in this appendix to electoral formulae that allocate seats on a proportional basis in multiple districts, calculate  $W_M$  for each district and add the results together.

Finally, note that votes are only wasted when a party does not gain representation from its fractional remainder. For example, if a party deserves 3.4 of 5 seats but only receives 3, the votes that contributed to the 0.4 remainder would be wasted. The fraction of the votes that this represents can be calculated in the following way:  $\frac{\text{fractional remainder}}{\text{total number of seats}} = \frac{0.4}{5} = 0.08$ . However, if this party had received 4 seats instead of 3, none of the votes cast for the party would have been wasted.

## **A.1 THE NUMBER OF PARTIES IS LESS THAN TWICE THE NUMBER OF SEATS**

### **A.1.1 THE NUMBER OF PARTIES IS EVEN**

To begin, take  $p$  to be 2. The most votes will be wasted when each party has a remainder of exactly half a seat (e.g., one party deserves 1.5 seats and the other 3.5). In order to assign all of the seats, one party will have to be over represented by half a seat and the other under represented by half a seat. The party that is under represented will be the source of the wasted votes. Therefore, in this case,  $\frac{0.5}{5} = 0.1 = 10\%$  of the votes will be wasted.

When  $p$  is 4, the same strategy will waste the most votes. Each party having a remainder of half a seat (e.g., three parties deserve 1.5 seats, and one party deserves 0.5) will waste  $\frac{1}{5} = 20\%$  of the votes. Similarly, when  $p$  is 6, 30% of the votes will be wasted with this strategy.

This strategy can be summarized as follows:

1. Divide the number of parties by 2.
2. Multiply by 1/2.
3. Divide by the number of seats.

This produces the following formula:

$$W_M = \frac{p}{2} \times \frac{1}{2} \times \frac{1}{s} = \frac{p}{4s} \quad p = 2, 4, 6, \dots, 2s$$

### A.1.2 THE NUMBER OF PARTIES IS ODD

The case where  $p$  is 1 will not be considered since it will be impossible to waste any of the votes. Therefore, the case where  $p$  equals 3 will be the starting point. To waste the most votes, the remainders for the parties need to be made as equal to each other and as close to  $1/2$  as possible. With three parties, this can be done in two ways: assign each party a fractional remainder of  $1/3$  or  $2/3$ . Both options produce the same result. For the sake of consistency with what is to follow, I will choose the first option. Since only one remainder seat needs to be assigned with this option, two parties will gain no representation from their remainders. As a result,  $2 \times \frac{1}{3} \times \frac{1}{5} = 13\%$  of the votes will be wasted.

When  $p$  is 5, the most votes will be wasted when each party has a fractional remainder of  $2/5$ . This will result in three parties receiving no representation from their remainders, which will waste  $3 \times \frac{2}{5} \times \frac{1}{5} = 24\%$  of the votes. Similarly, when  $p$  is 7,  $4 \times \frac{3}{7} \times \frac{1}{5} = 34\%$  of the votes will be wasted.

This strategy can be summarized as follows:

1. Determine the fractional remainder of each party:
  - a. Subtract 1 from the total number of parties.
  - b. Divide by 2.
  - c. Divide by the total number of parties.
2. Multiply the result from 1 by the number of parties plus one.

3. Divide by 2.
4. Divide by the total number of seats.

In formulaic form, this becomes:

$$W_M = \frac{p-1}{2p} \times \frac{p+1}{2s} = \frac{p^2-1}{4ps} \quad p = 1, 3, 5, 7, \dots, 2s-1$$

## A.2 THE NUMBER OF PARTIES IS GREATER THAN TWICE THE NUMBER OF SEATS

Once the number of parties exceeds twice the number of seats, the strategies applied in Section A.1 will no longer work. The reason is that, in these situations, there are not enough seats to create the seat distributions made earlier. One can also determine that the above formulae will not hold for all  $p$  greater than one by observing that in the limit that  $p$  approaches infinity, both formulae also approach infinity. Since it is impossible for  $W_M$  to be greater than unity, these formulae do not hold for sufficiently large  $p$ . Therefore, one test for the formula that will be derived in this section is that it approach one as  $p$  approaches infinity.

To maximize the number of wasted votes, we again want to make the fractional remainders of the parties equal to one another and as close to 1/2 as possible. When the number of parties is greater than twice the number of seats, this is easy to achieve. First, divide the number of seats by the number of parties to determine the fraction of a seat each party will deserve. Then, multiply this fraction by the number of parties minus the number of seats (this is the number of parties that will not receive representation). Finally, to determine the fraction of the votes that this represents, divide by the number of seats. Here is this process in formulaic form:

$$W_M = \frac{s}{p} \times \frac{p-s}{s} = 1 - \frac{s}{p} \quad p \geq 2s$$

It can be easily observed that in the limit where  $p$  approaches infinity,  $W_M$  approaches 1 as required.

## APPENDIX B: BALLOT DESIGN

<b>Dual-Member Mixed Proportional Ballot: Option A</b>	
<input type="checkbox"/>	Red Party <ol style="list-style-type: none"><li>1. Candidate A</li><li>2. Candidate B</li></ol>
<input type="checkbox"/>	Blue Party <ol style="list-style-type: none"><li>1. Candidate A</li><li>2. Candidate B</li></ol>
<input type="checkbox"/>	Orange Party <ol style="list-style-type: none"><li>1. Candidate A</li><li>2. Candidate B</li></ol>

**Dual-Member Mixed  
Proportional Ballot:  
Option B**

1 2 3 4 5 6 7 8 9 10    Red Party

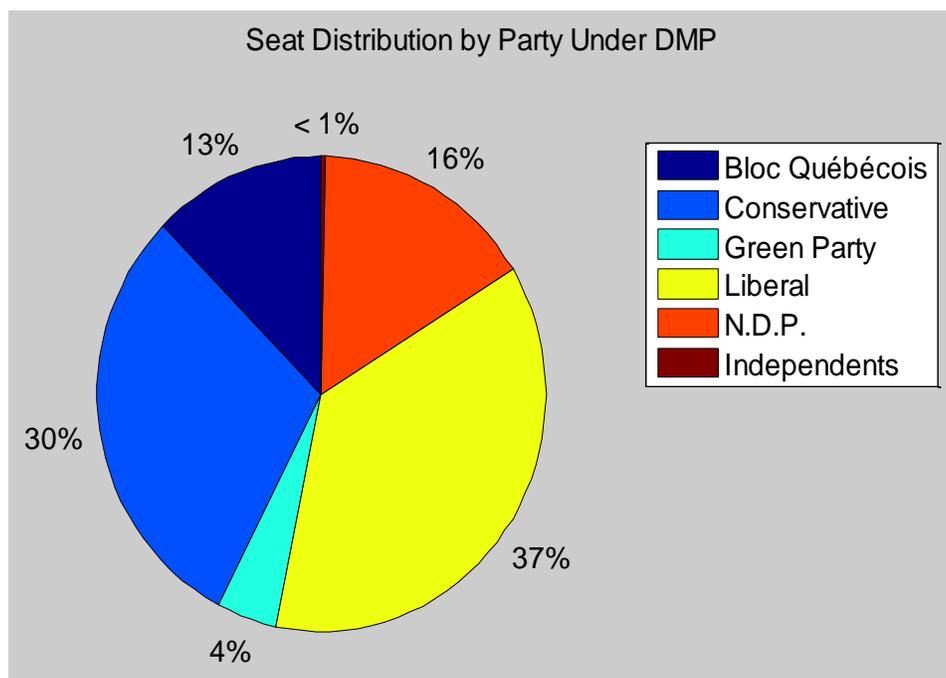
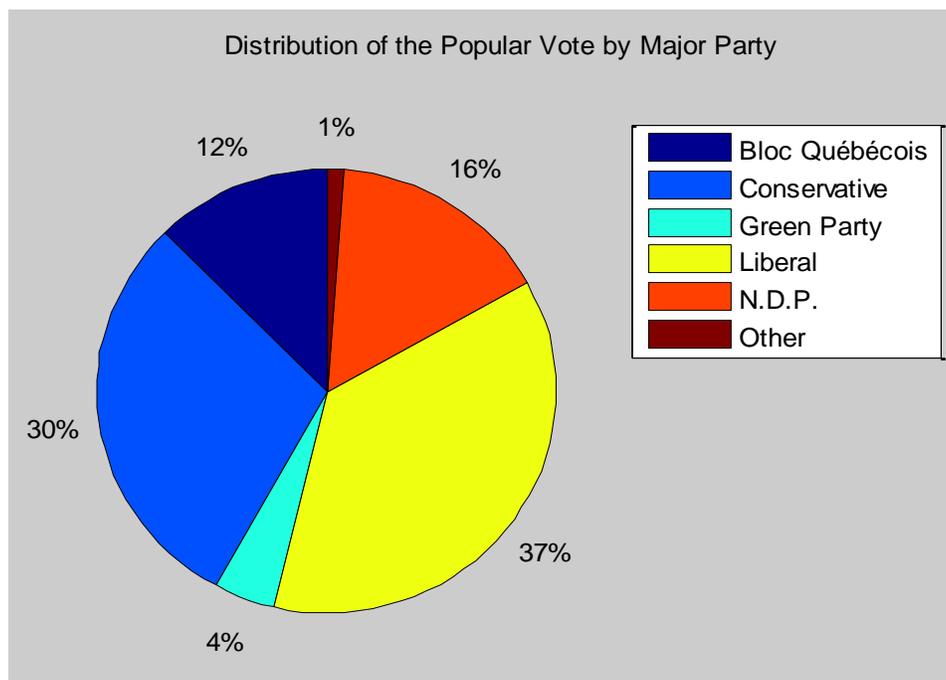
1. Candidate A
2. Candidate B

1 2 3 4 5 6 7 8 9 10    Blue Party

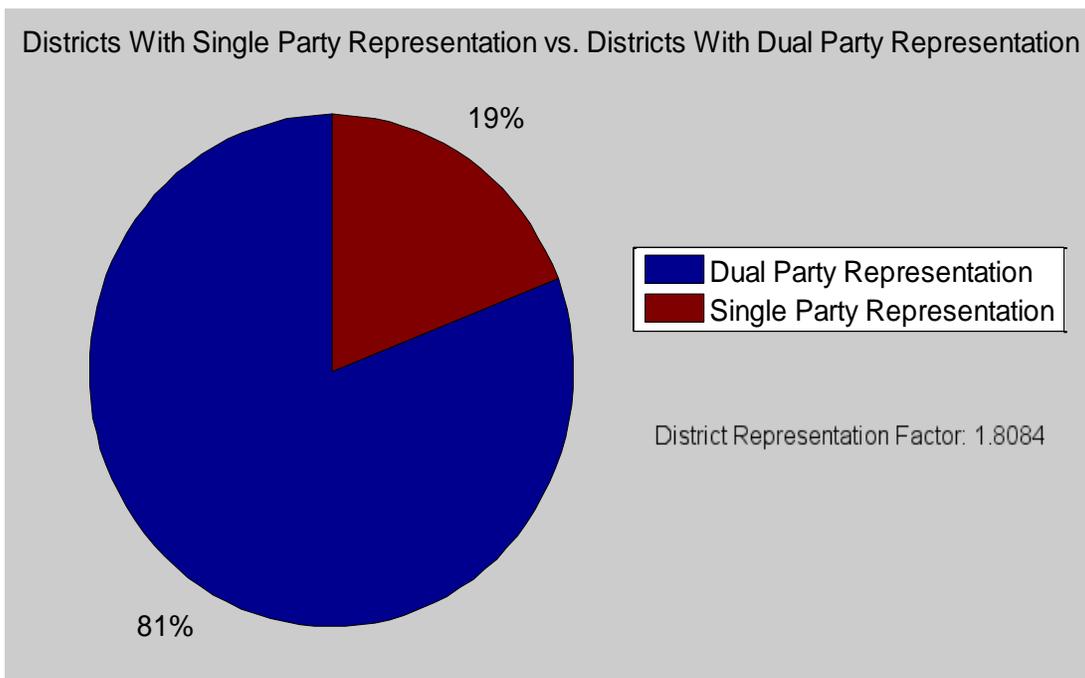
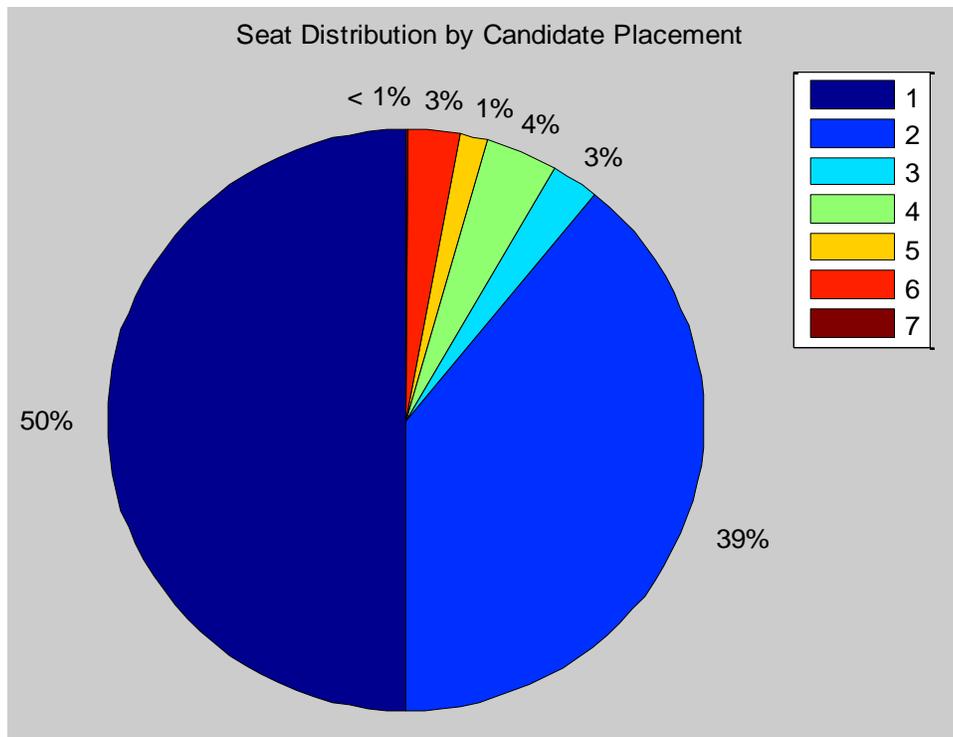
1. Candidate A
2. Candidate B

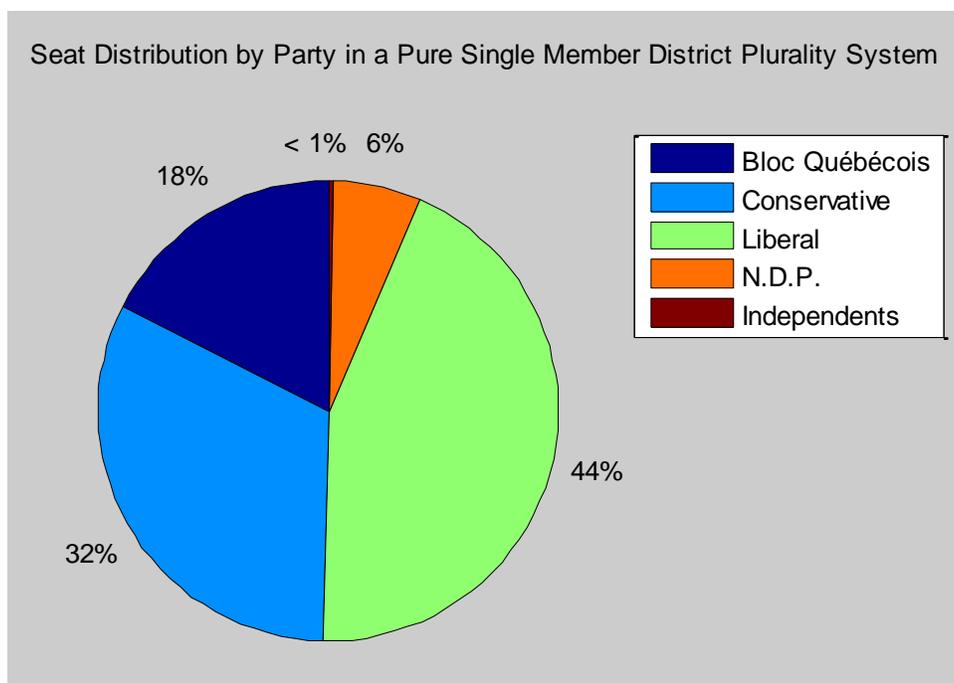
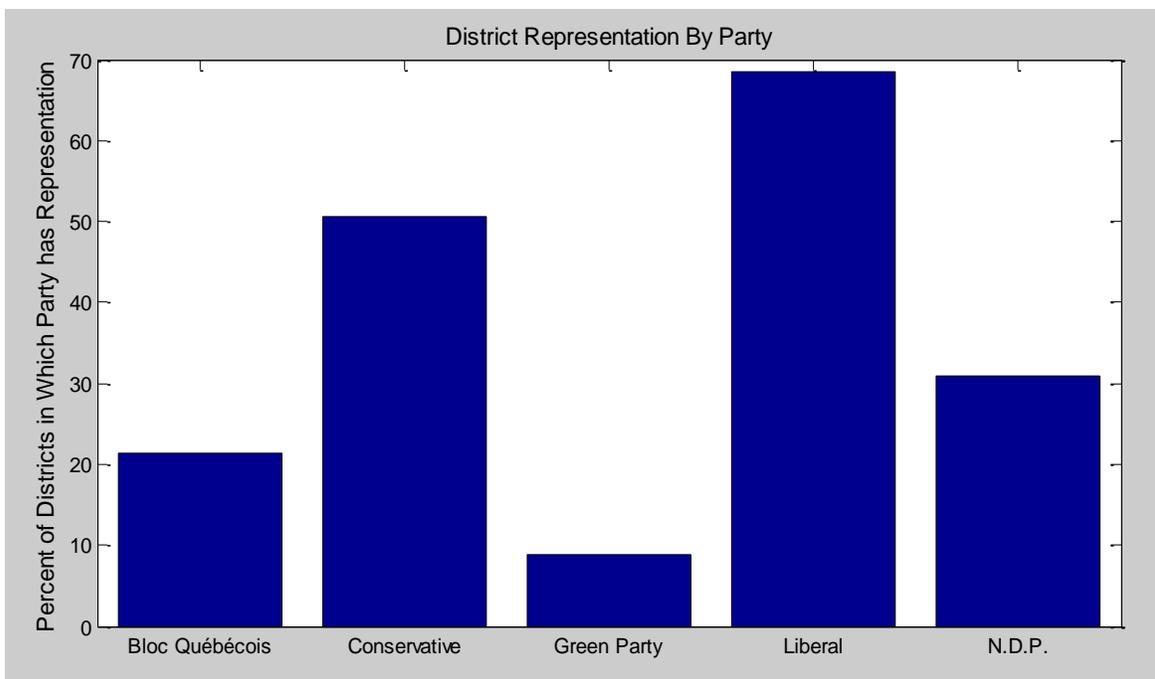
1 2 3 4 5 6 7 8 9 10    Orange Party

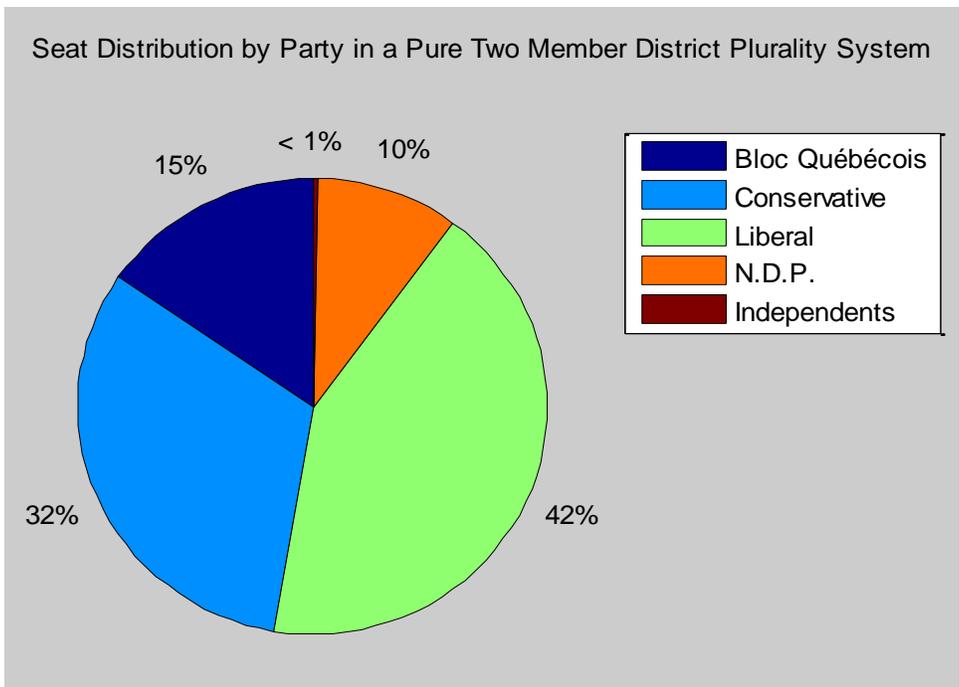
1. Candidate A
2. Candidate B

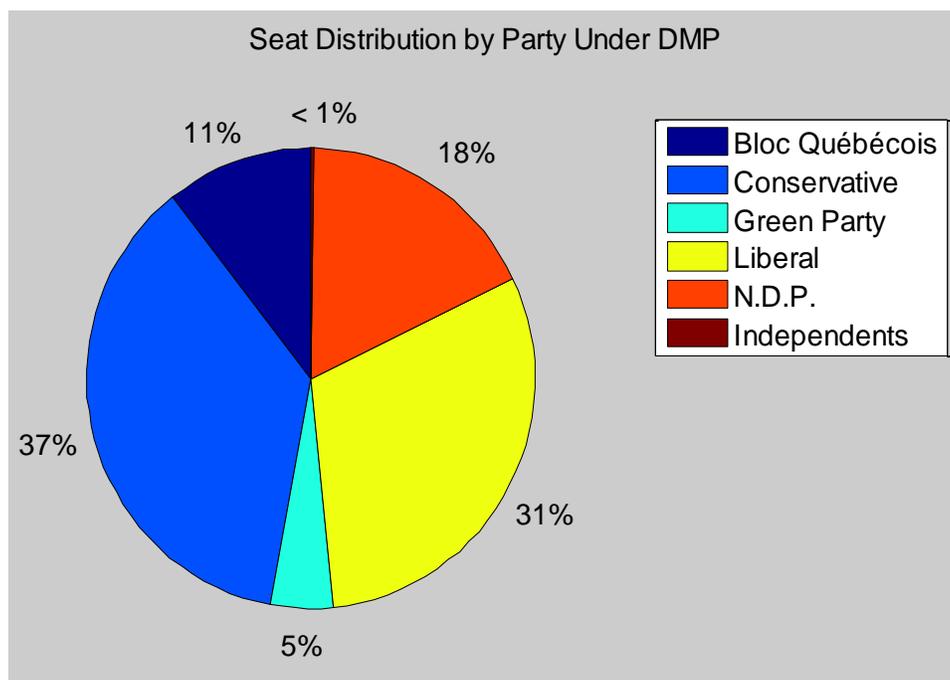
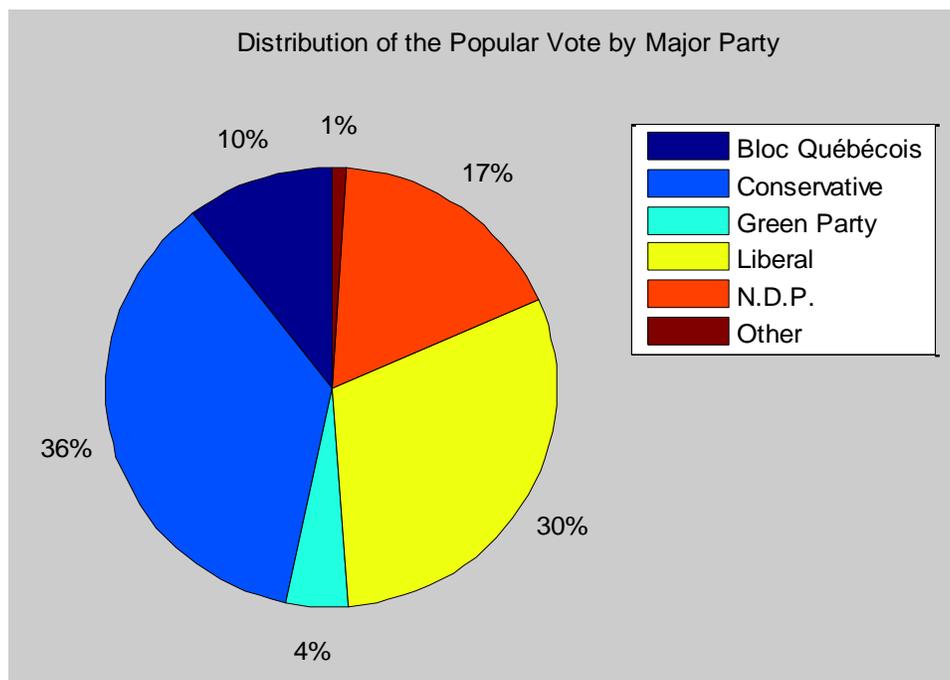
APPENDIX C: 2004 FEDERAL ELECTION<sup>83</sup>

<sup>83</sup> “38<sup>th</sup> General Election—Poll-by-Poll Results—Raw Data,” Table 12, last modified August 24, 2010, [http://www.elections.ca/scripts/resval/ovr\\_2004.asp?prov=&lang=e](http://www.elections.ca/scripts/resval/ovr_2004.asp?prov=&lang=e). The charts in this appendix have been produced by running the raw election data in Table 12 through a collection of MATLAB files.

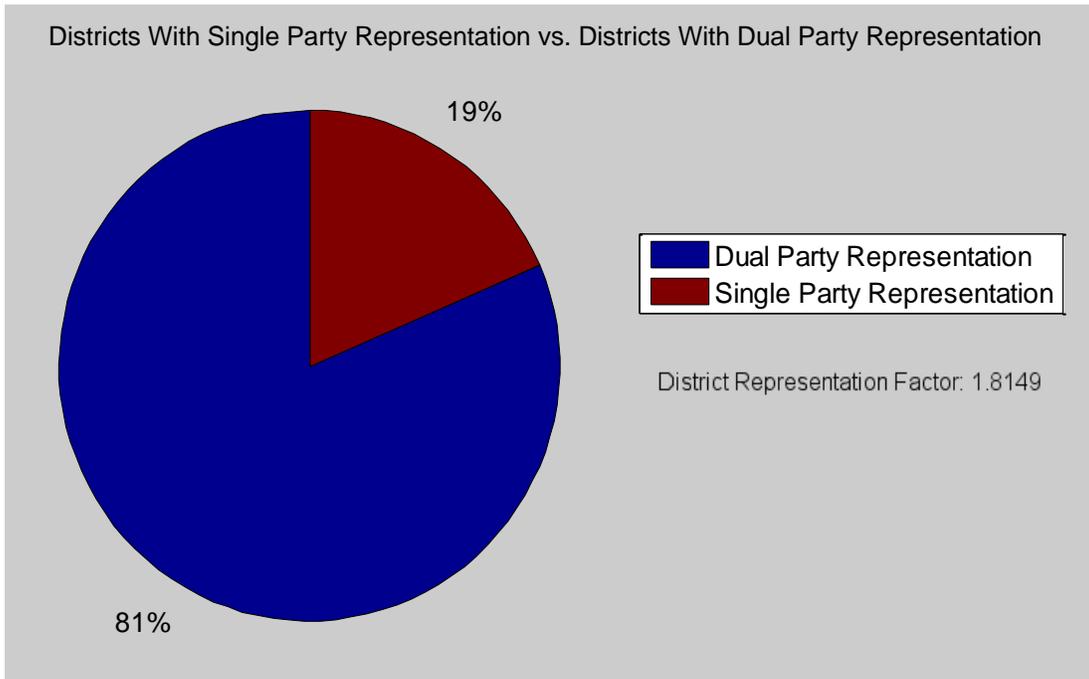
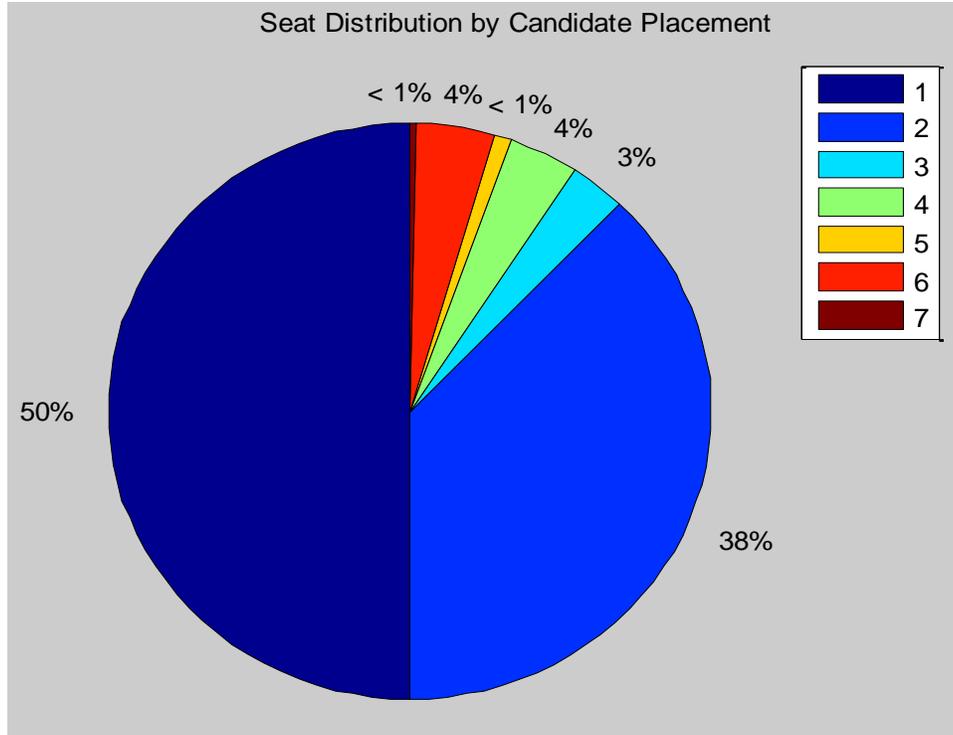


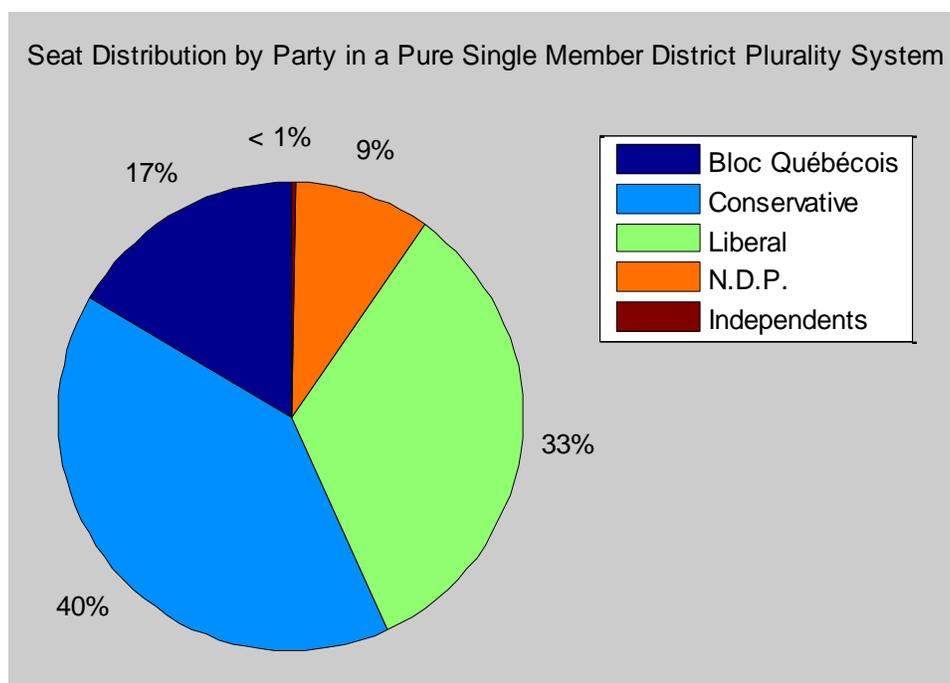
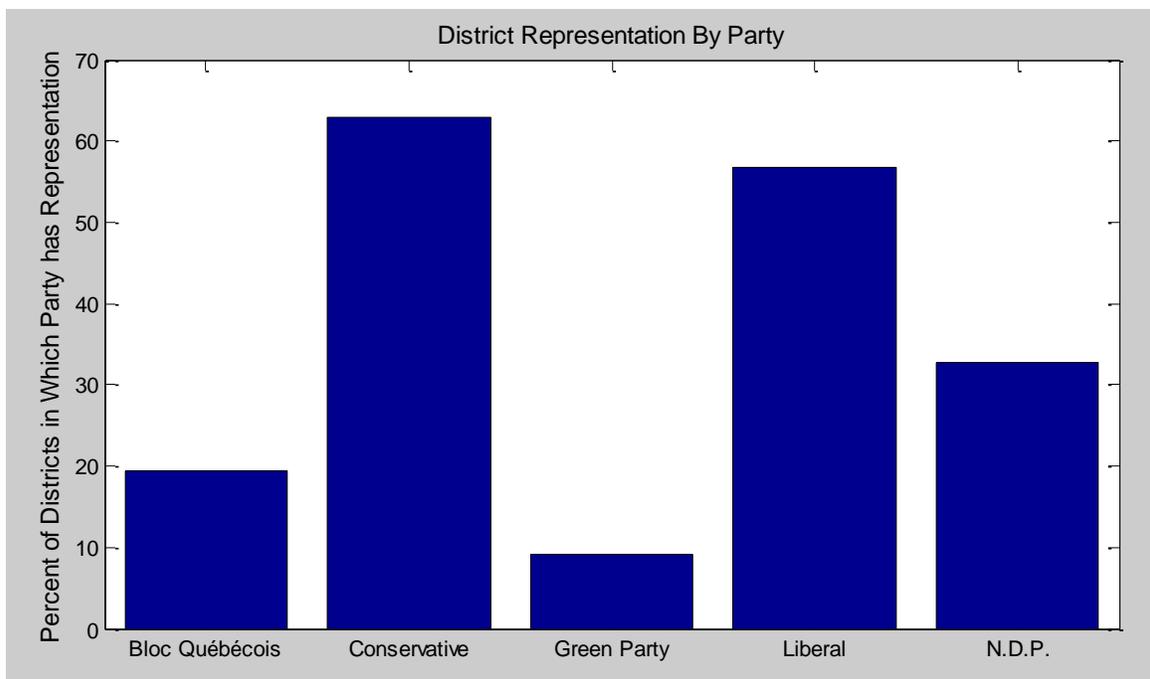


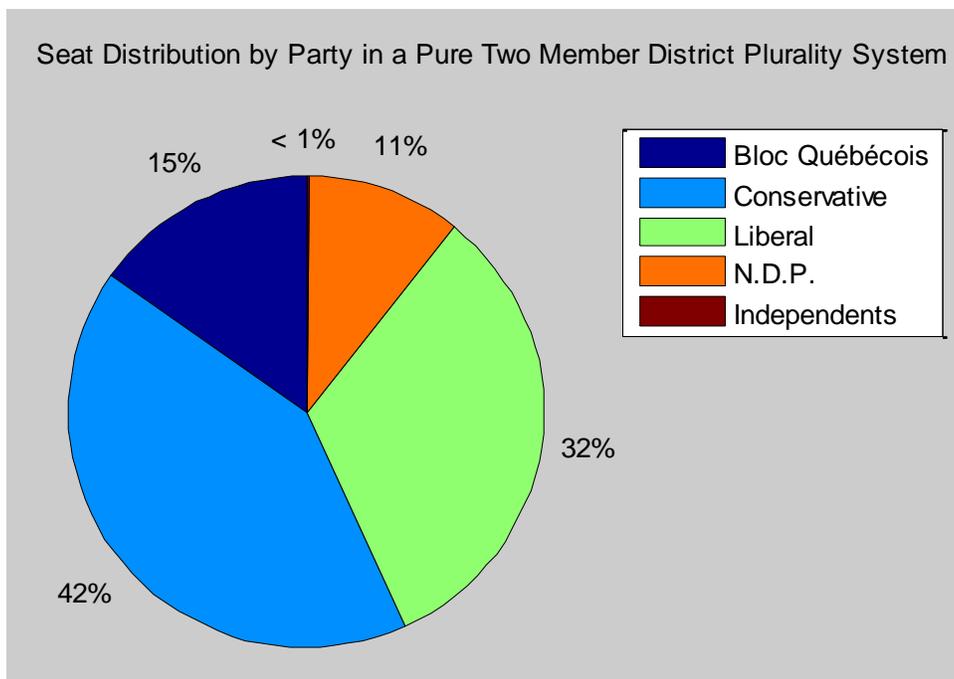


APPENDIX D: 2006 FEDERAL ELECTION<sup>84</sup>

<sup>84</sup> “39<sup>th</sup> General Election—Poll-by-Poll Results—Raw Data,” Table 12, last modified August 24, 2010, [http://www.elections.ca/scripts/resval/ovr\\_39ge.asp?prov=&lang=e](http://www.elections.ca/scripts/resval/ovr_39ge.asp?prov=&lang=e). The charts in this appendix have been produced by running the raw election data in Table 12 through a collection of MATLAB files.

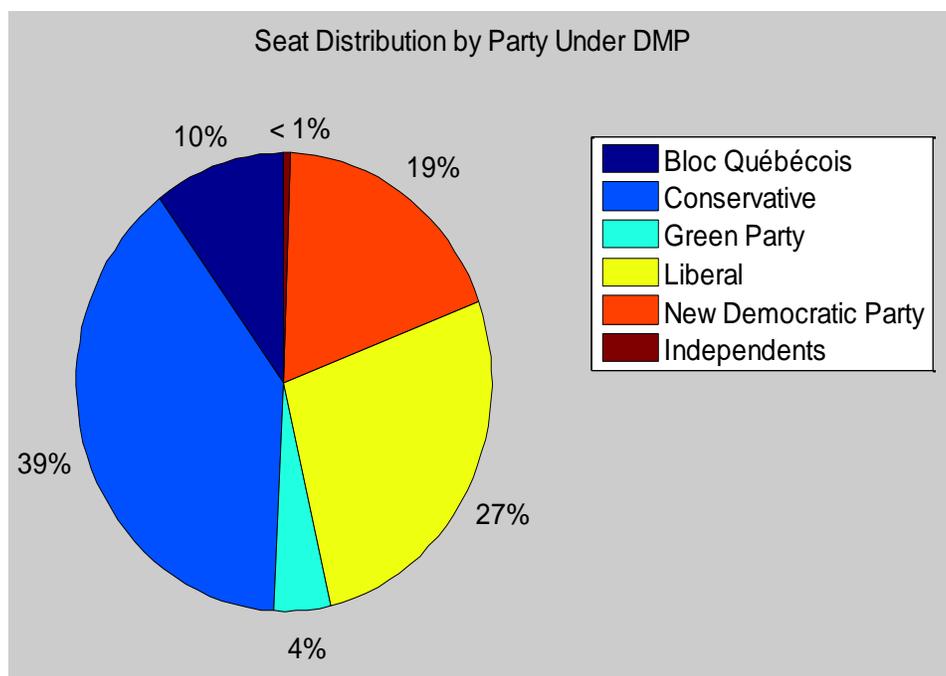
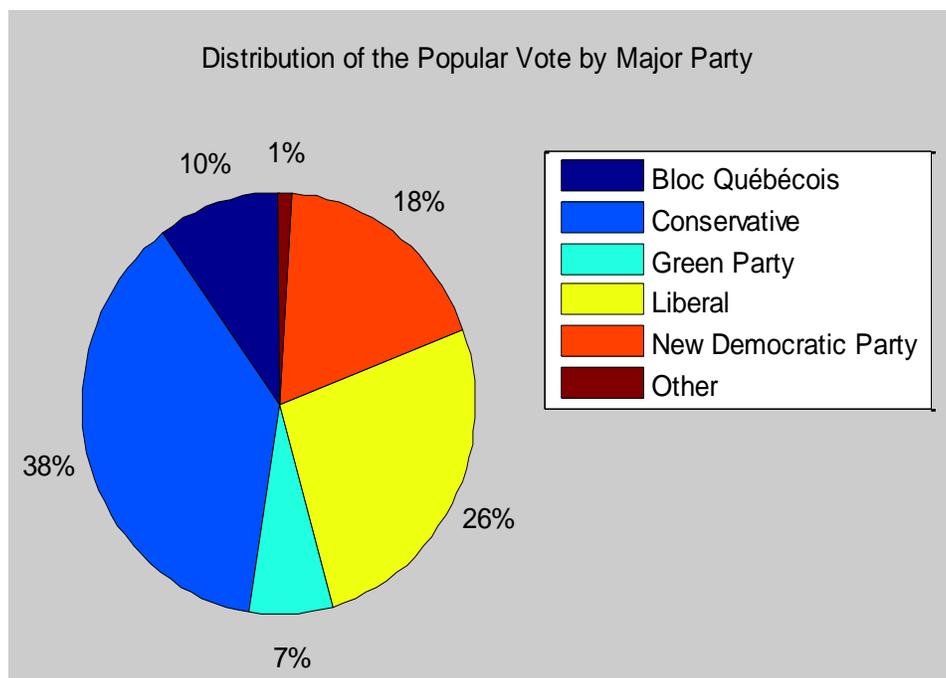




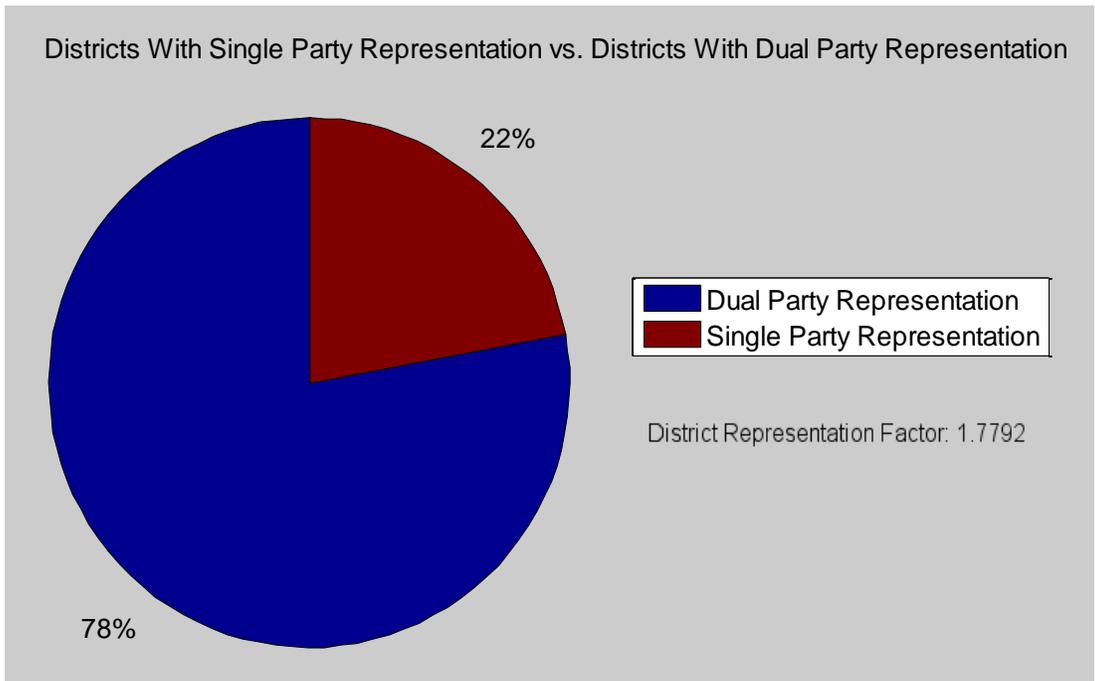
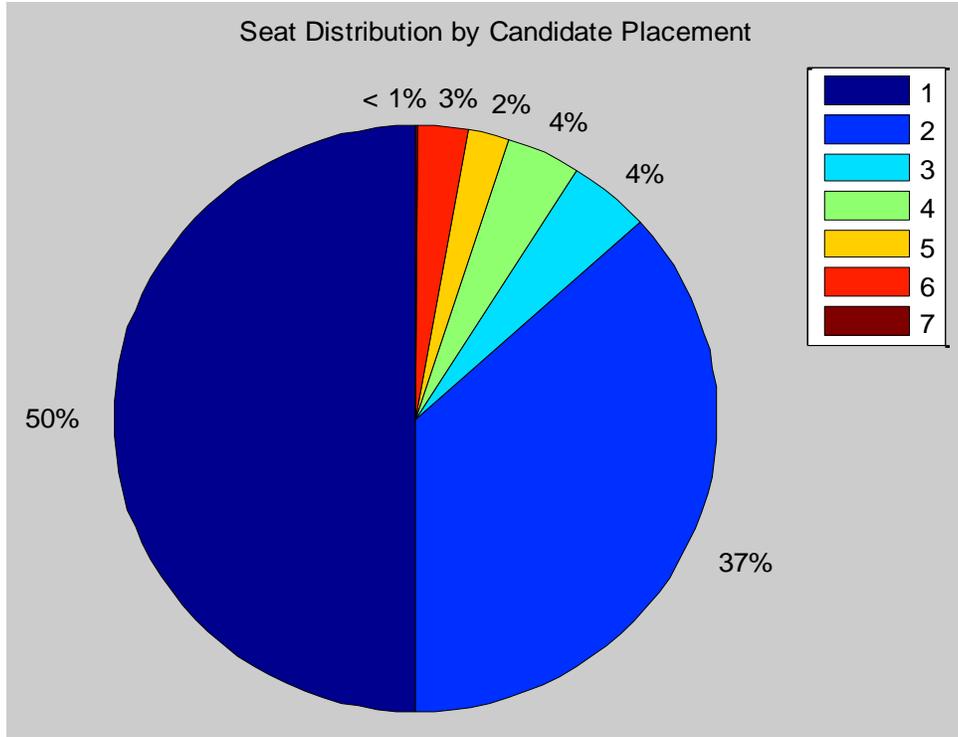


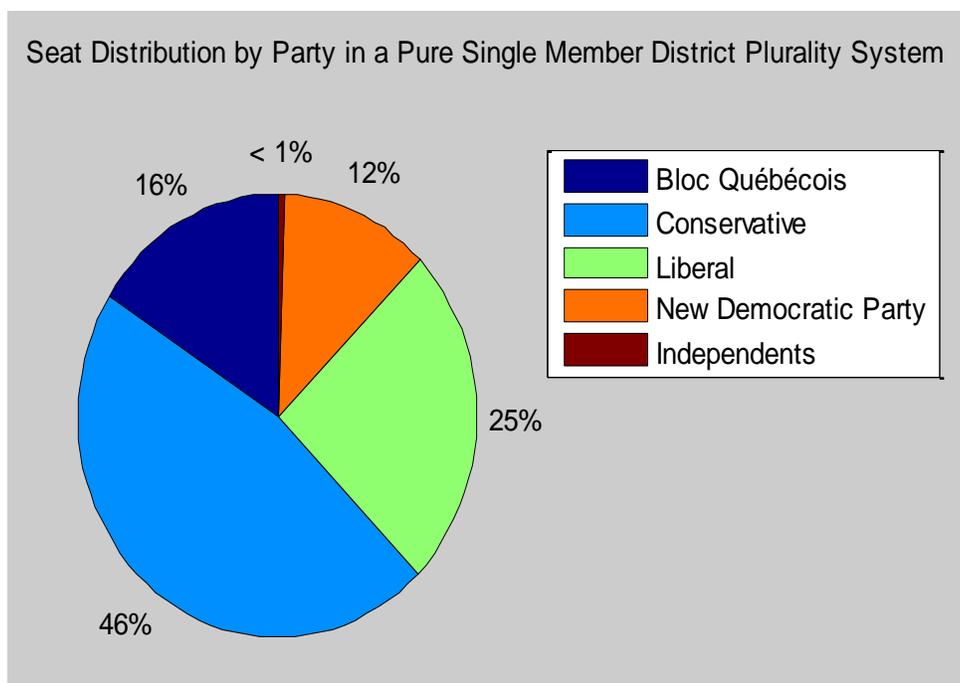
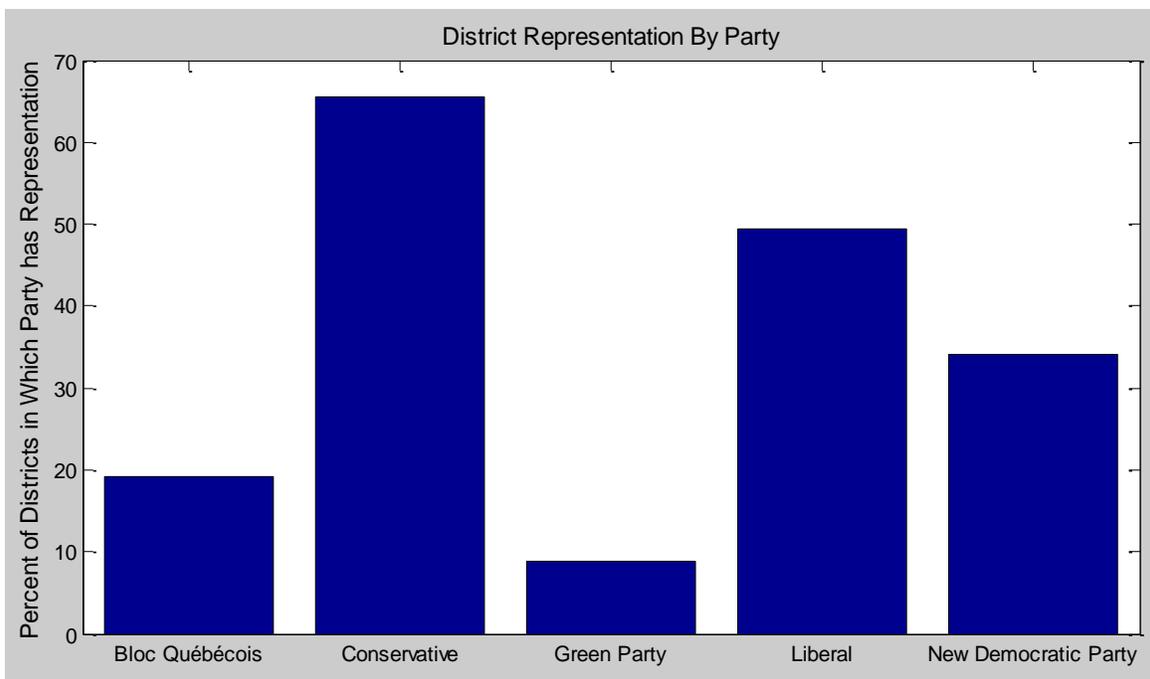
APPENDIX E: 2008 FEDERAL ELECTION<sup>85</sup>

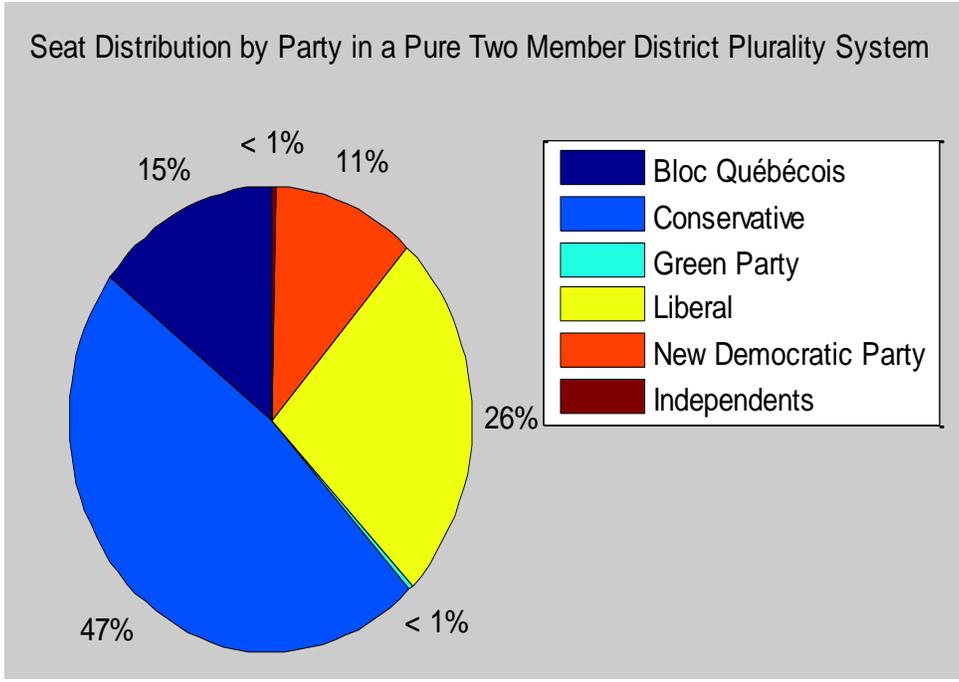
## E.1 FEDERAL RESULTS



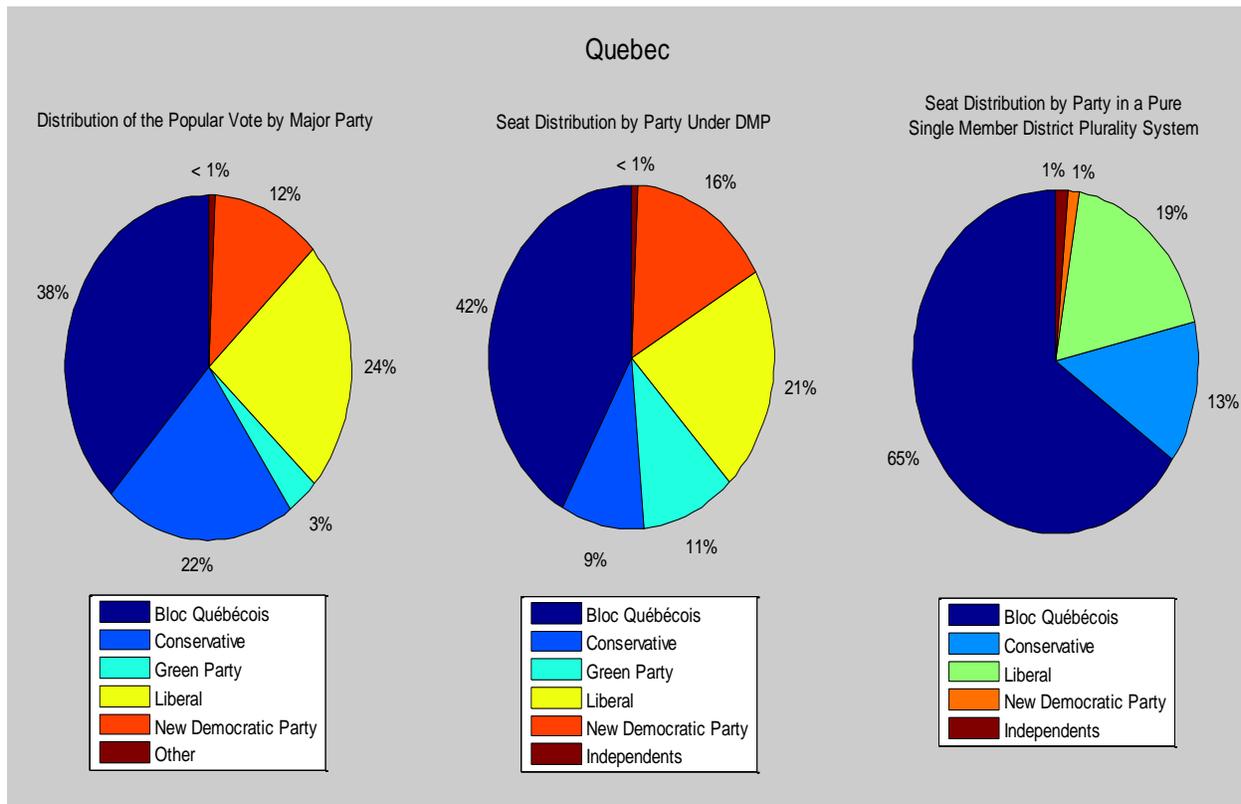
<sup>85</sup> “40<sup>th</sup> General Election—Poll-by-Poll Results—Raw Data,” Table 12, last modified August 24, 2010, [http://www.elections.ca/scripts/resval/ovr\\_40ge.asp?prov=&lang=e](http://www.elections.ca/scripts/resval/ovr_40ge.asp?prov=&lang=e). The charts in this appendix have been produced by running the raw election data in Table 12 through a collection of MATLAB files.





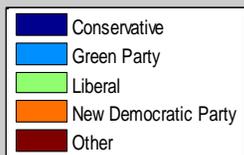
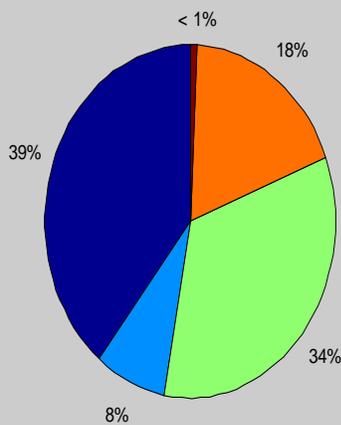


**E.2 SELECT PROVINCIAL RESULTS**

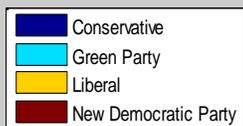
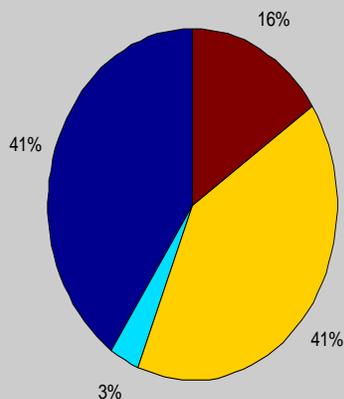


### Ontario

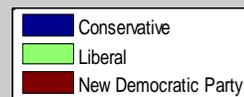
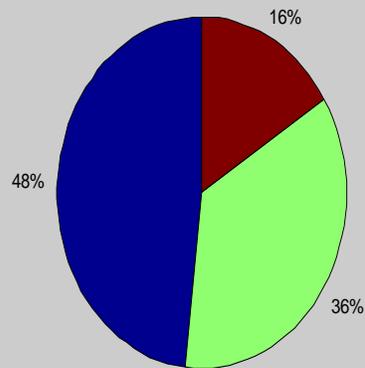
Distribution of the Popular Vote by Major Party



Seat Distribution by Party Under DMP

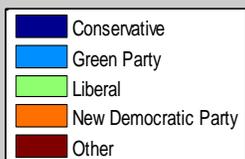
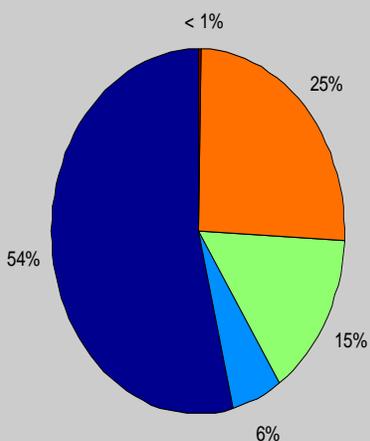


Seat Distribution by Party in a Pure Single Member District Plurality System

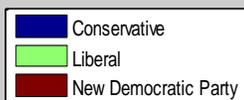
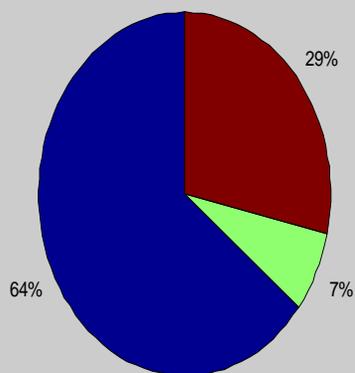


### Saskatchewan

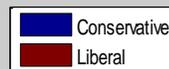
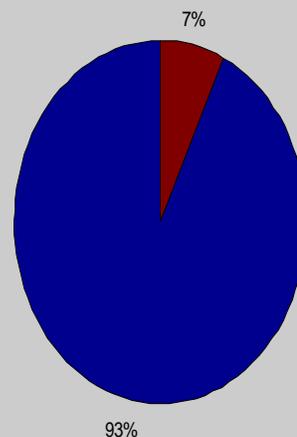
Distribution of the Popular Vote by Major Party



Seat Distribution by Party Under DMP

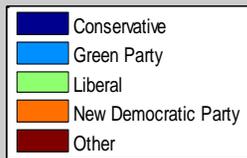
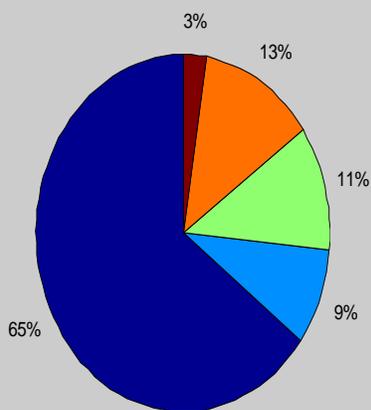


Seat Distribution by Party in a Pure Single Member District Plurality System

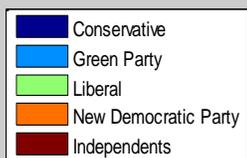
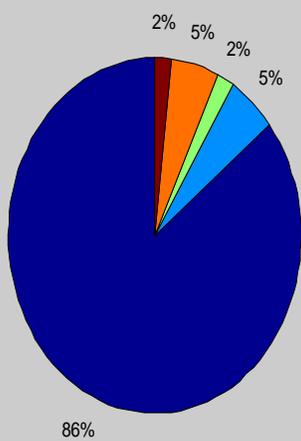


### Alberta

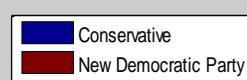
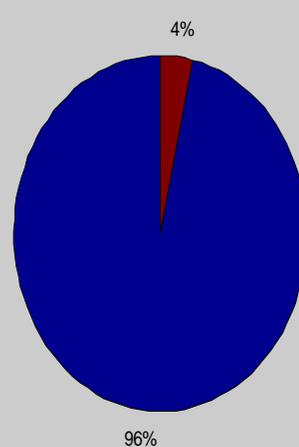
Distribution of the Popular Vote by Major Party



Seat Distribution by Party Under DMP

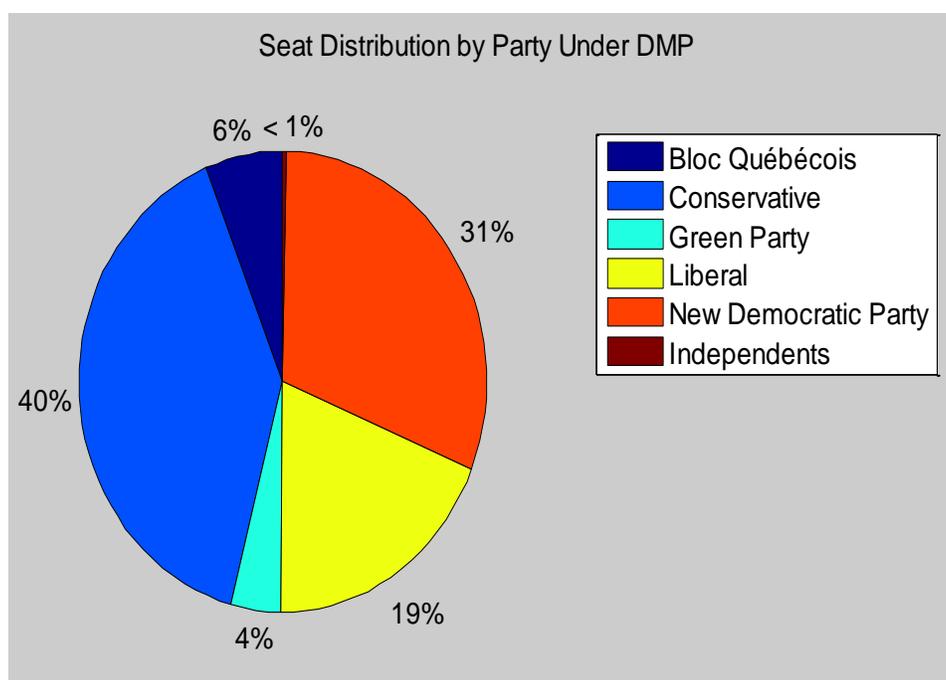
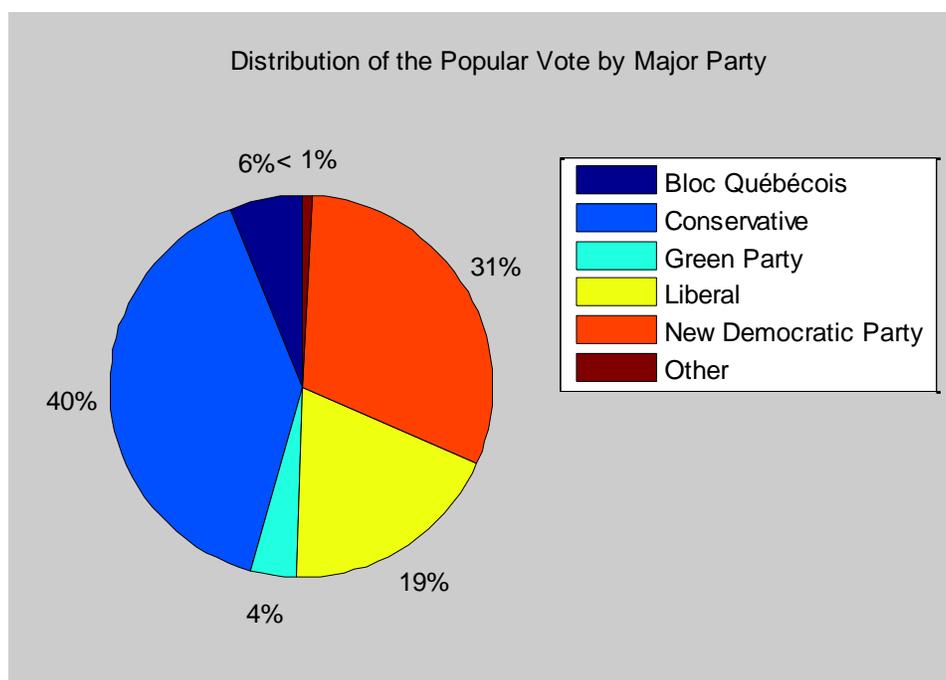


Seat Distribution by Party in a Pure Single Member District Plurality System

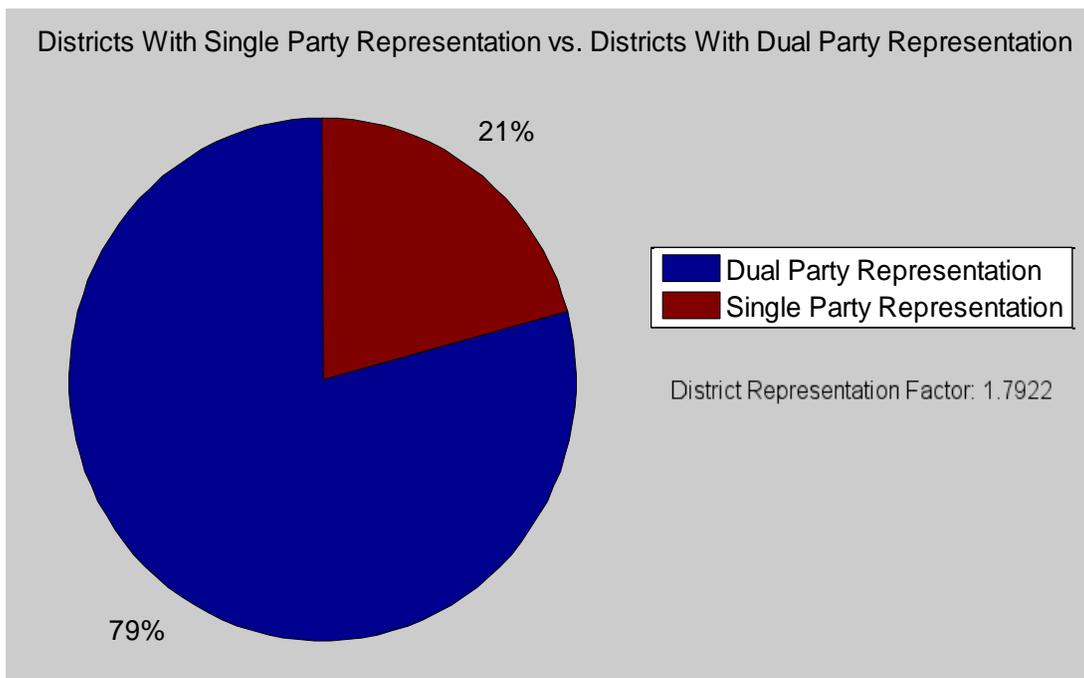
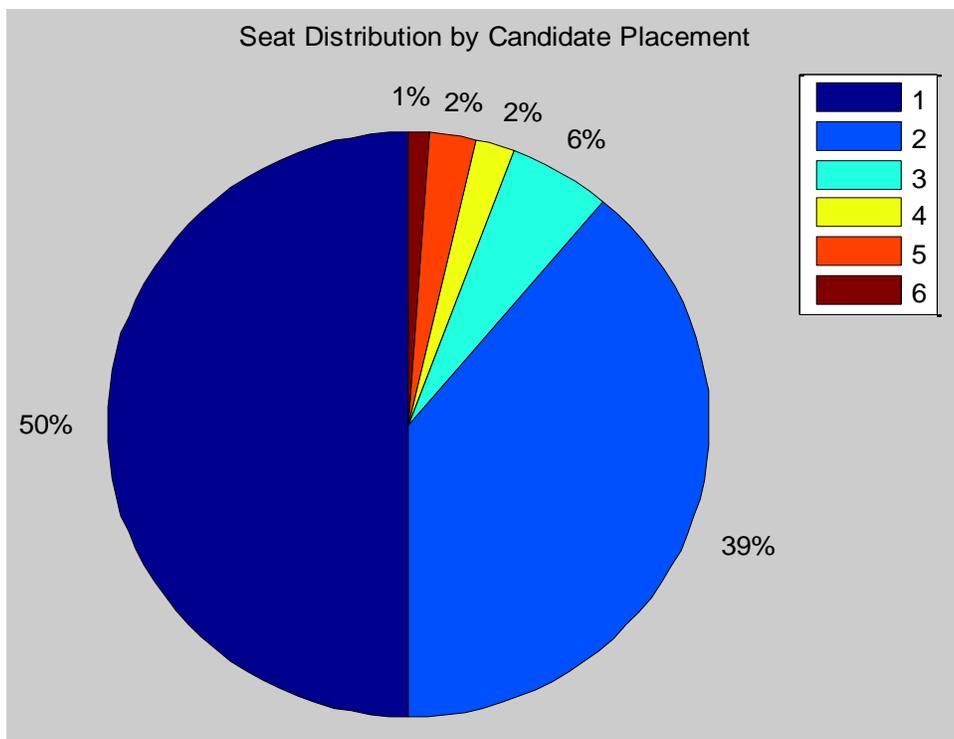


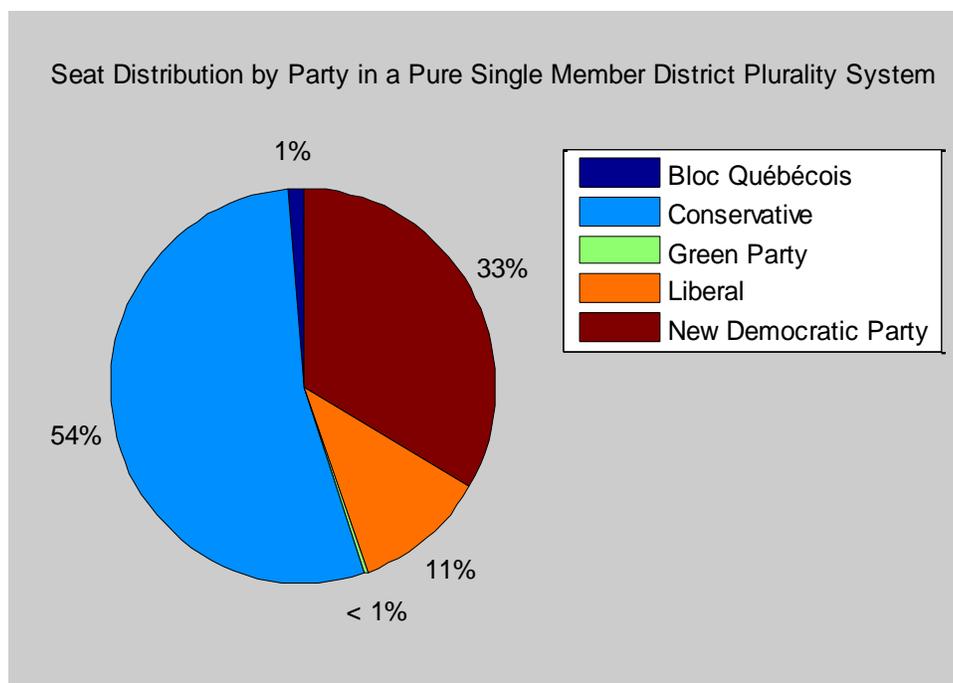
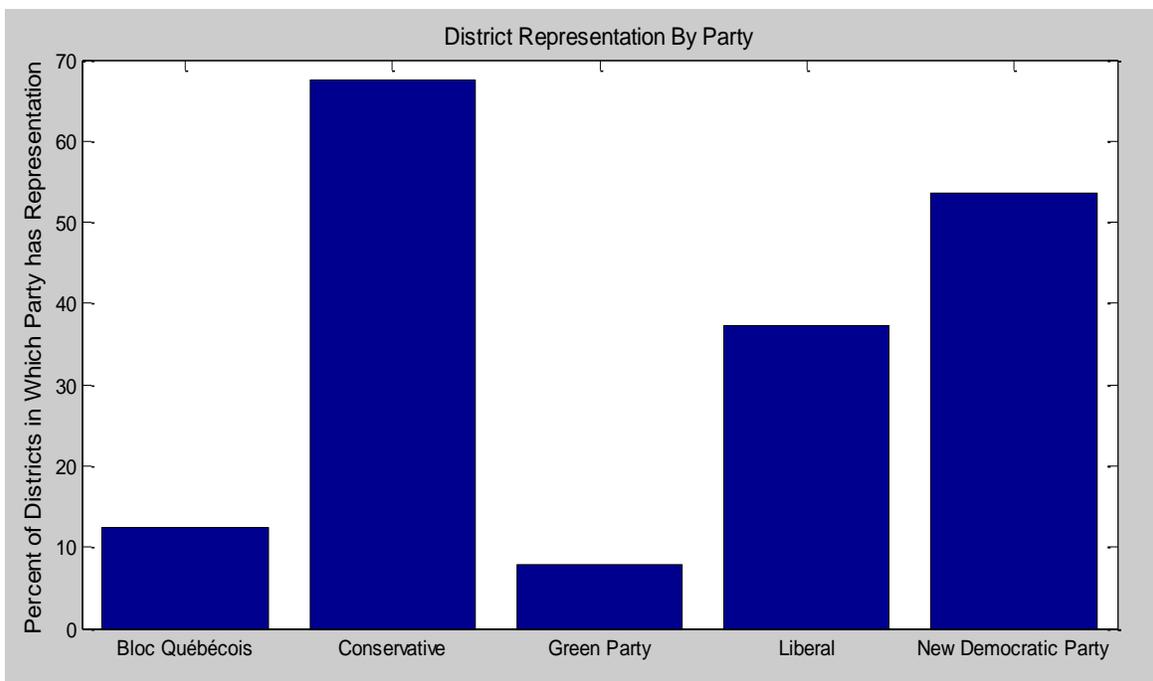
APPENDIX F: 2011 FEDERAL ELECTION<sup>86</sup>

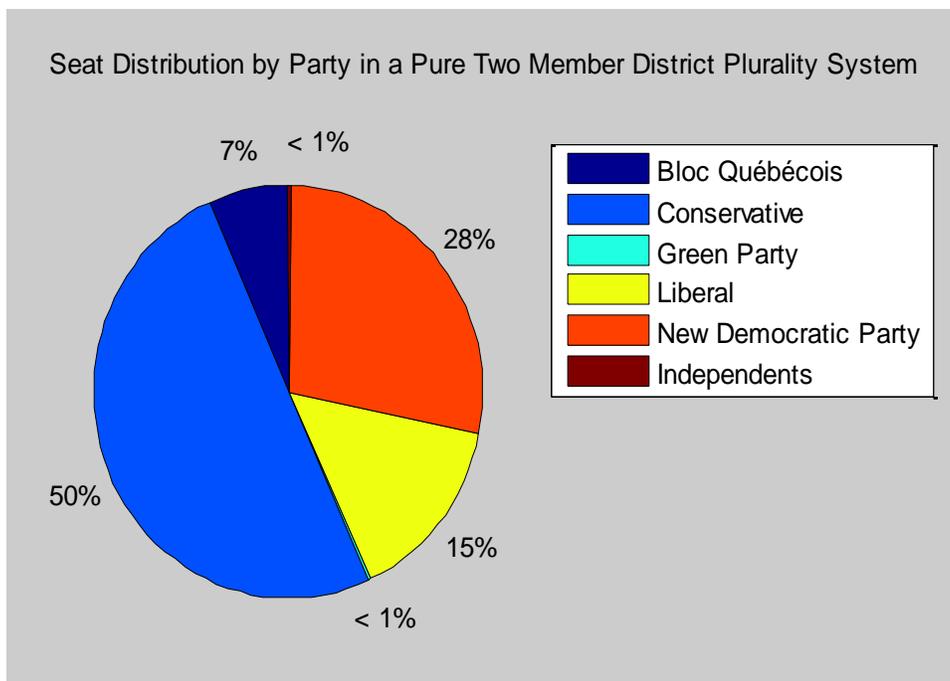
## F.1 FEDERAL RESULTS



<sup>86</sup> “41<sup>st</sup> General Election—Poll-by-Poll Results—Raw Data,” Table 12, last modified January 10, 2013, [http://www.elections.ca/scripts/resval/ovr\\_41ge.asp?prov=&lang=e](http://www.elections.ca/scripts/resval/ovr_41ge.asp?prov=&lang=e). The charts in this appendix have been produced by running the raw election data in Table 12 through a collection of MATLAB files.

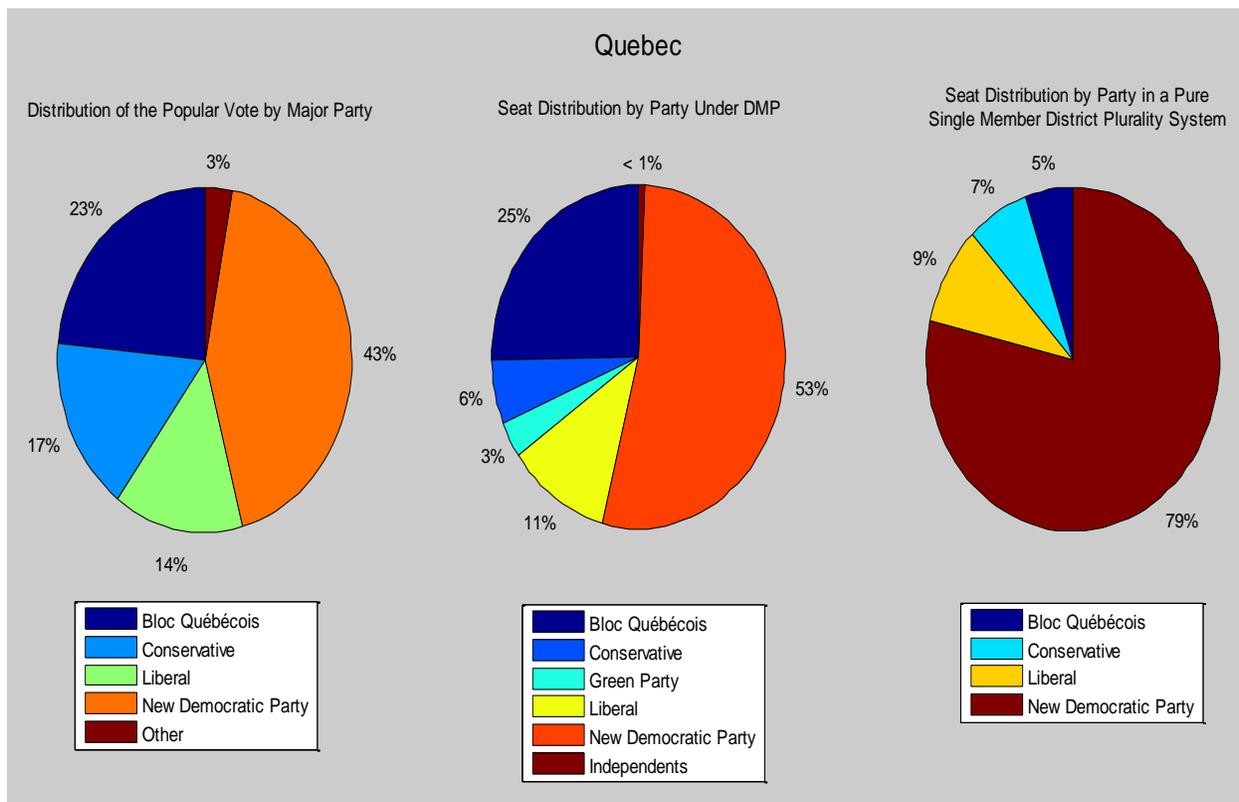






87

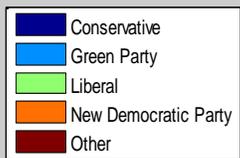
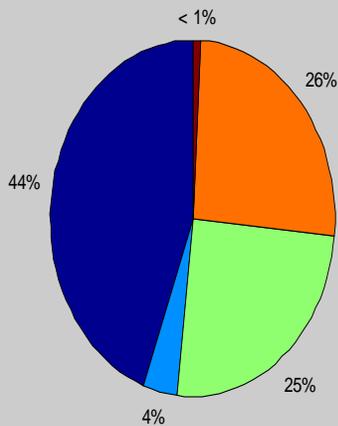
## F.2 SELECT PROVINCIAL RESULTS



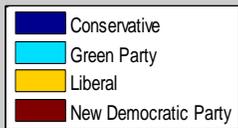
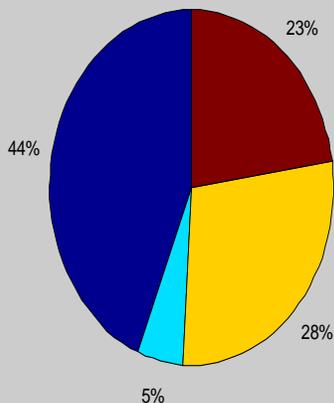
<sup>87</sup> The seat count for the Conservative Party here is 309 of 616 seats.

### Ontario

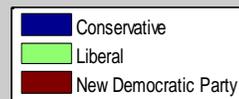
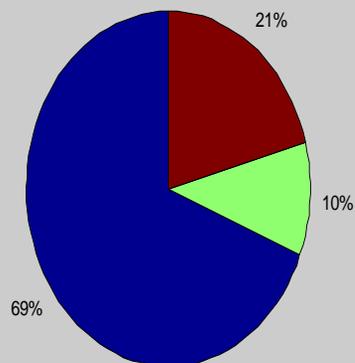
Distribution of the Popular Vote by Major Party



Seat Distribution by Party Under DMP

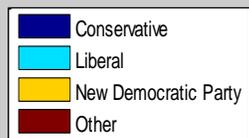
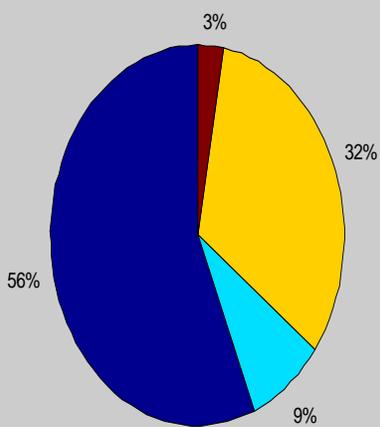


Seat Distribution by Party in a Pure Single Member District Plurality System

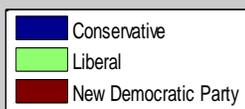
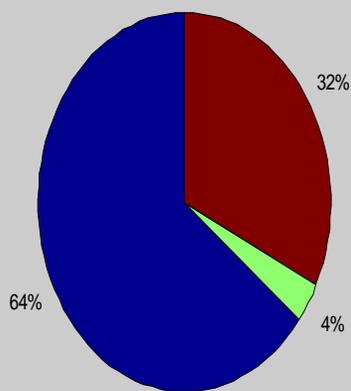


### Saskatchewan

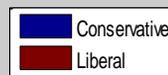
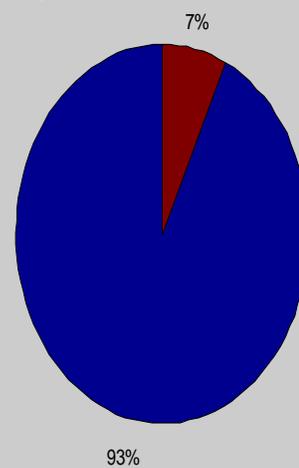
Distribution of the Popular Vote by Major Party



Seat Distribution by Party Under DMP

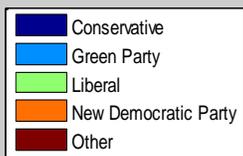
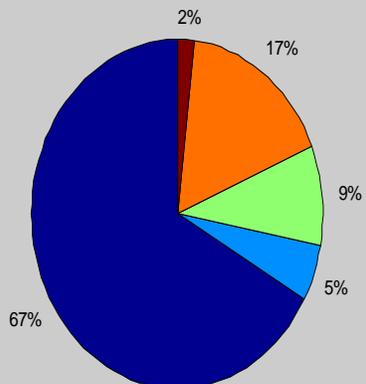


Seat Distribution by Party in a Pure Single Member District Plurality System

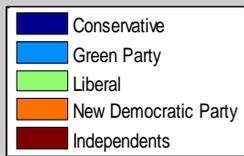
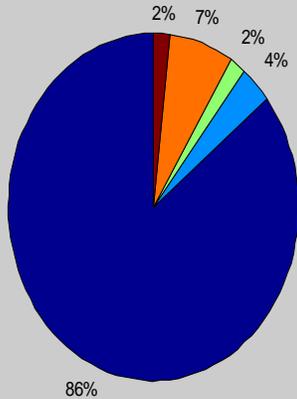


### Alberta

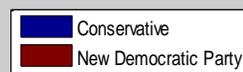
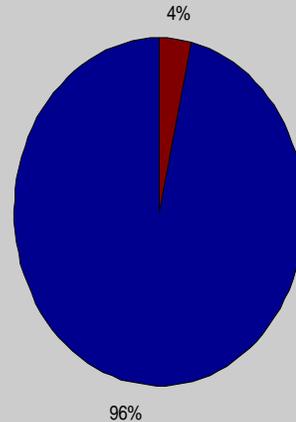
Distribution of the Popular Vote by Major Party



Seat Distribution by Party Under DMP



Seat Distribution by Party in a Pure Single Member District Plurality System



## APPENDIX G: THE DISTRICT REPRESENTATION FACTOR

The district representation factor is the number that when multiplied by a party's fraction of the seats in an elected assembly yields the fraction of the districts in which the party has representation. In an electoral system with single-member districts, the district representation factor is always one. Since DMP makes use of two-member districts, the district representation factor can range from one to two. If a party were to win at most one seat per district, its district representation factor would be two. Conversely, if a party were to win either two or zero seats in each district, its district representation factor would be one. Since it is possible for parties to win one seat in some districts and two seats in others, the district representation factor can take on values between one and two. The purpose of this appendix is to derive a formula to calculate the district representation factor when two-member districts are in use. In the discussion that follows,  $T_p$  will represent the total number of seats a party has won,  $T_s$  will represent the total number of seats,  $T_d$  will represent the total number of districts,  $D$  will represent the number of seats a party has won in districts with dual party representation, and  $S$  will represent the number of seats a party has won in districts with single party representation.

The first step is to determine the number of districts each seat a party has won represents. When a party wins one seat in a district, the seat adds one district to the total districts in which the party is represented. When a party wins both seats in a district, each seat counts for only half a district. To calculate the number of districts each seat adds to a party's total, the effect of the two situations is averaged. The following formula shows this calculation:

$$\text{districts/seat} = \frac{D + S/2}{T_p}$$

The next step is to determine the fraction of districts in which a party has representation. This is done by multiplying the above result with the number of seats a party has won and dividing by the total number of districts. Finally, the district representation factor is calculated by dividing the fraction of the districts a party is represented in by its fraction of the seats. This yields the following:

$$\text{District Representation Factor} = \frac{\frac{(D + S/2)}{T_p} \times \frac{T_p}{T_d}}{\frac{T_p}{T_s}} = \frac{(D + S/2)}{T_p} \times \frac{T_s}{T_d}$$

This result can be simplified further since the relationship between the number of seats and the number of districts is fixed. There are always twice as many seats as there are districts; therefore, the district representation factor can be expressed as follows:

$$\text{District Representation Factor} = \frac{2D + S}{T_p}$$

The formula derived in this section is not needed to calculate the district representation factor for a party. Instead, it is needed to calculate the district representation factor based on the fraction of districts with dual and single party representation. By assuming that the proportions of seats each party has in districts with dual and single party representation is the same as the overall distribution, a general district representation factor can be calculated. Therefore, the district representation factors that have been calculated for the past four federal elections are not specific to any one party. Although these district representation factors are reasonably accurate for some parties, they are only intended to provide a measure of the overall effect.

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